Algebra 2

Ch. 5 Handout 5.6

Complex Numbers

Number Systems

1. Is there a solution to the equation x + 1 = 0 on the system of natural numbers? $x+1=0$, $x=-1$

2. Is there a solution to the equation 2x – 1 = 0 on the system of integers?

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

answer

1. The solution of this equation is x = -1, which is a integer but not a natural number.
2. The solution of this equation is x = 1/2, which is a rational number not a integer.

Number Systems

3. Is there a solution to the equation $x^2 - 2 = 0$ on the system of rational numbers?

$x^2 - 2 = 0$

$\sqrt{x^2} = \pm\sqrt{2}$

$x = \pm\sqrt{2}$

4. Is there a solution to the equation $x^2 + 1 = 0$ on the system of real numbers?

$x^2 + 1 = 0$

$\sqrt{x^2} = \pm\sqrt{-1}$

$x = \pm\sqrt{-1}$

answer

3. The solution of this equation is $x = \pm\sqrt{2}$, which is a irrational number but not a rational number.

4. The solution of this equation is $x = \pm\sqrt{-1}$, which is not a real number. Also explain that these types of numbers fall in a set known as Complex Numbers.

The imaginary number **i** is defined as the number whose squares is -1.

$$i = \sqrt{-1}$$

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$i^3 = i^2 \cdot i$

$= -1 \cdot i$

$\boxed{i^3 = -i}$

$i^4 = i^2 \cdot i^2$

$= (-1)(-1)$

$\boxed{i^4 = 1}$

OR $i^4 = (i^2)^2 = (-1)^2 = 1$

$i^6 = (i^2)^3$

$= (-1)^3$

$\boxed{i^6 = -1}$

$i^9 = i^8 \cdot i$

$= (i^2)^4 \cdot i$

$= (-1)^4 = i$

$= 1 \cdot i$

$\boxed{i^9 = i}$

$i^{20} = (i^2)^{10}$

$= (-1)^{10}$

$\boxed{i^{20} = 1}$

$i^{31} = i^{30} \cdot i$

$= (i^2)^{15} \cdot i$

$= (-1)^{15} \cdot i$

$= -1 \cdot i$

$\boxed{i^{31} = -i}$

An **imaginary number** is any number of the form $a + bi$, where a and b are real numbers, and $b \neq 0$.

Real # Imaginary #

Square Root of a Negative Real Number

For any positive real number a, $\sqrt{-a} = i\sqrt{a}$.

Example: $\sqrt{-4} = \sqrt{-1 \cdot 4} = i\sqrt{4} = i \cdot 2 = 2i$

Note that $\left(\sqrt{-4}\right)^2 = \left(i\sqrt{4}\right)^2 = i^2\left(\sqrt{4}\right)^2 = -1 \cdot (2)^2 = -1 \cdot 4 = -4$

Simplifying Imaginary Numbers.

1) $\sqrt{-8}$

$\sqrt{-1 \cdot 8}$

$\sqrt{-1 \cdot 4 \cdot 2}$

$\sqrt{-1}\ \sqrt{4}\ \sqrt{2}$

$\boxed{2i\sqrt{2}}$

2) $\sqrt{-54}$

$\sqrt{-1 \cdot 54}$

$\sqrt{-1 \cdot 9 \cdot 6}$

$\sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{6}$

$\boxed{3i\sqrt{6}}$

3) $-\sqrt{-169} = -\sqrt{-1 \cdot 169} = -\sqrt{-1} \cdot \sqrt{169}$

$= \boxed{-13i}$

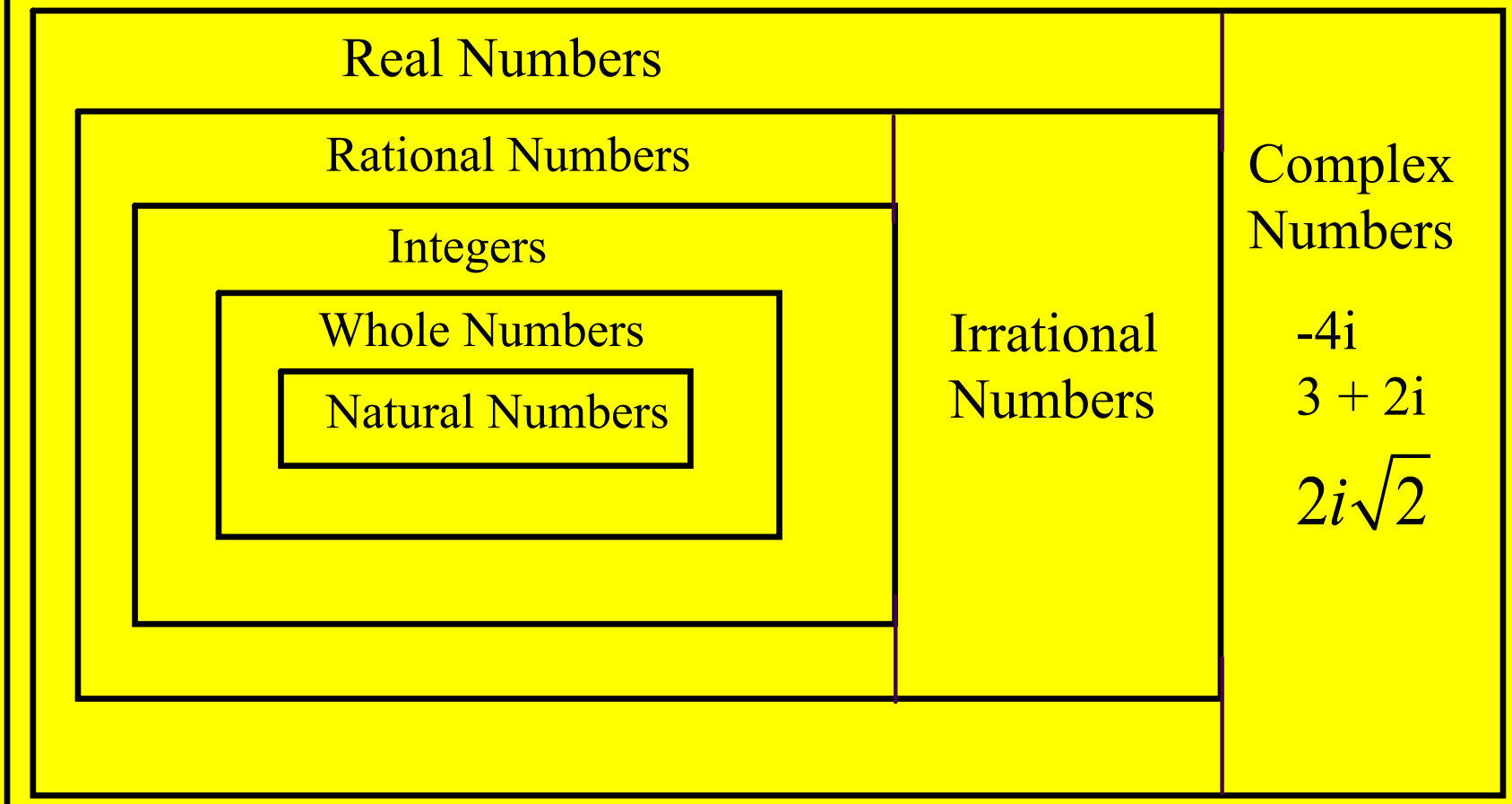
Complex Number System

Complex Numbers

A complex number can be written in the form a + bi, where a and b are real numbers, including 0.

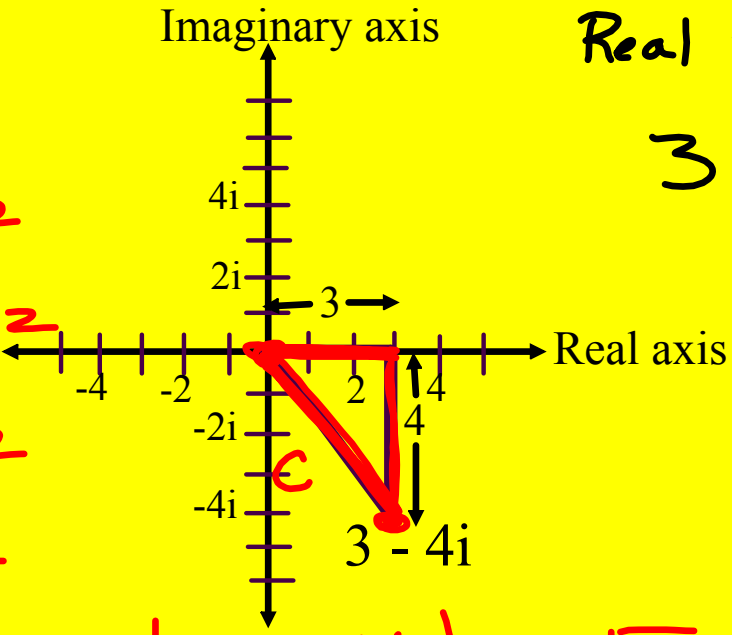
a + bi

real part imaginary part



Real # ± Imaginary #

$3 - 4i$



$a^2 + b^2 = c^2$

$(3)^2 + (4)^2 = c^2$

$9 + 16 = c^2$

$\sqrt{25} = \sqrt{c^2}$

$c = 5$

$|3 - 4i| = 5$

Simplifying Imaginary Numbers

4. Write the complex number $\sqrt{-121} - 7$ in the form of a + bi.

Real #

Imaginary #

$\sqrt{-1 \cdot 121} - 7$

$11i - 7$

$\boxed{-7 + 11i}$

5. Simplify each number by using the imaginary number

a) $\sqrt{-2}$

$\sqrt{-1 \cdot 2}$

$i\sqrt{2}$

b) $\sqrt{-12}$

$\sqrt{-1 \cdot 12}$

$\sqrt{-1 \cdot 4 \cdot 3}$

$\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3}$

$2i\sqrt{3}$

c) $\sqrt{-36}$

$\sqrt{-1 \cdot 36}$

$6i$

6. Write the complex number $\sqrt{-18}+7$ in the form a + bi.

$7+\sqrt{-1\cdot 18}$

$7+\sqrt{-1\cdot 9\cdot 2}$

$\boxed{7+3i\sqrt{2}}$

7. Find each absolute value.

b) $|10+24i|$ (R, I)

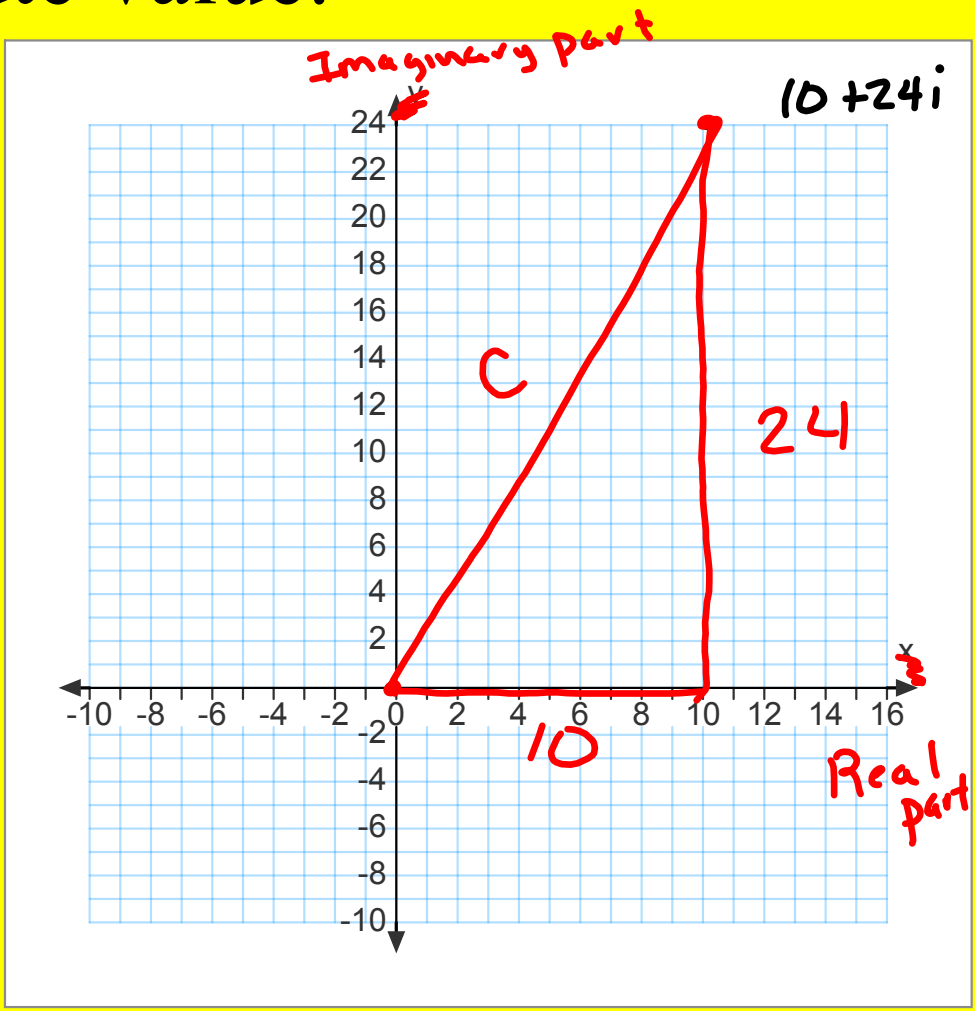
$10^2 + 24^2 = c^2$

$100 + 576 = c^2$

$\sqrt{c^2} = \sqrt{676}$

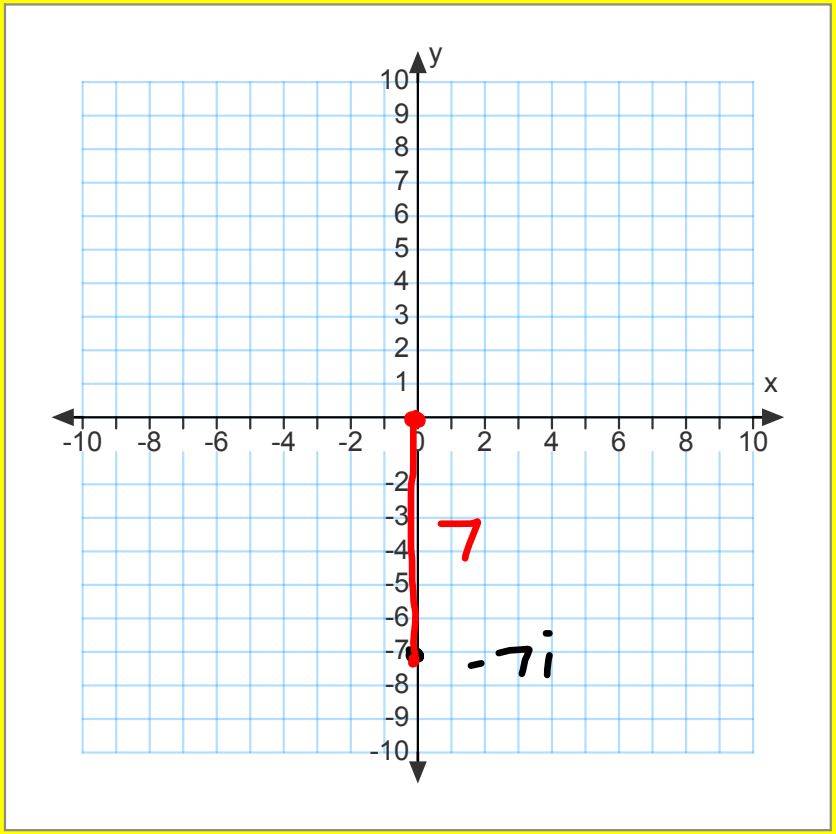
$c = 26$

$|10+24i| = 26$



7. Find each absolute value.

a) $|-7i| = 7$



Find the additive inverse of each number.

a) 3i

$-(3i)$

$-3i$

b) $-4+5i$

$-(-4+5i)$

$4-5i$

c) $-7-8i$

$-(-7-8i)$

$7+8i$

Assignment:

Day 1: pgs 278-280 1-28, 54, 55, 76, 80