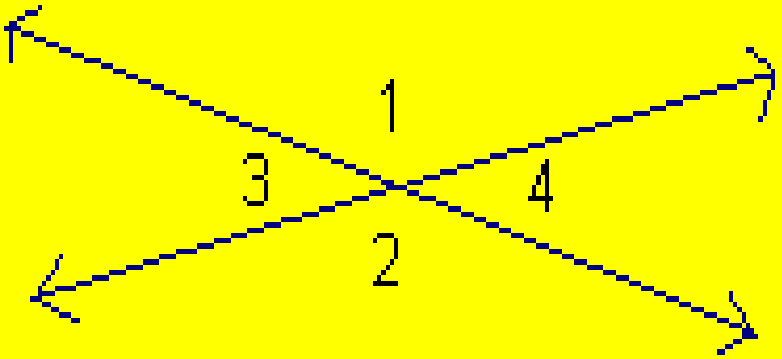
Geometry

Ch. 2 Handout 2.5

Proving Angles Congruent

Vertical Angles

two angles whose sides are opposite rays
(look for intersecting lines).



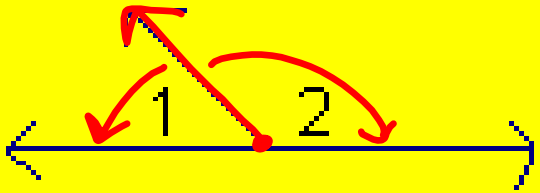
∠1 is vert. ∠2

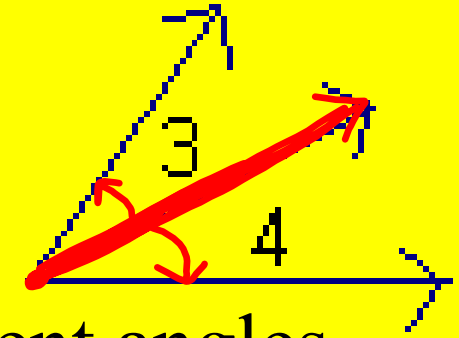
∠3 is vert. ∠4

Pull

Name 2 pairs of vertical angles:

Adjacent Angles-- two coplanar angles with a common side, a common vertex, and no interior points.



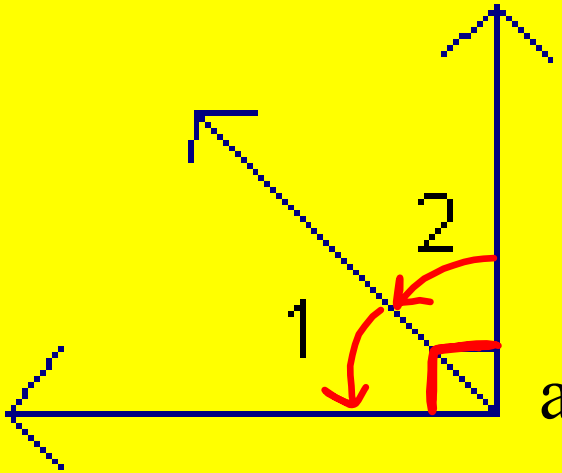


∠1 and ∠2 are adjacent angles as are ∠3 and ∠4.

Complementary Angles:

two angles whose measures have sum 90. Each angle is called the complement of the other.

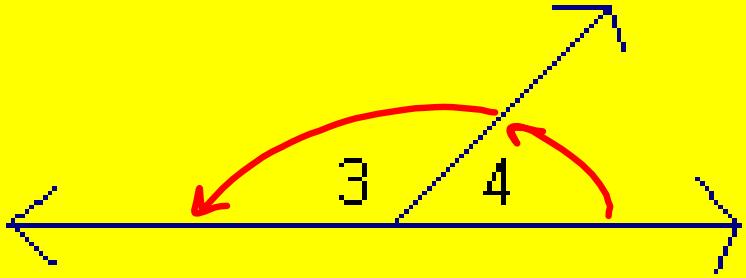
Pull



$\angle 1$ and $\underline{\quad \angle 2 \quad}$ are complementary angles because $m\angle 1 + m\angle 2 = 90$.

Supplementary Angles:

two angles whose measures have the sum of 180. Each angle is called the supplement of the other.



Pull

$\angle 3$ and $\angle 4$ are supplementary angles because $m\angle 3 + m\angle 4 = 180$

A proof is a logical argument that shows why a statement is true (or perhaps false).

We will be writing proofs in two-column format. This method allows you to see both the mathematical steps and their justifications side-by-side. Each statement in the proof must have a reason.

How to Write a Proof

~ List the given information

~ Use the information from the diagram

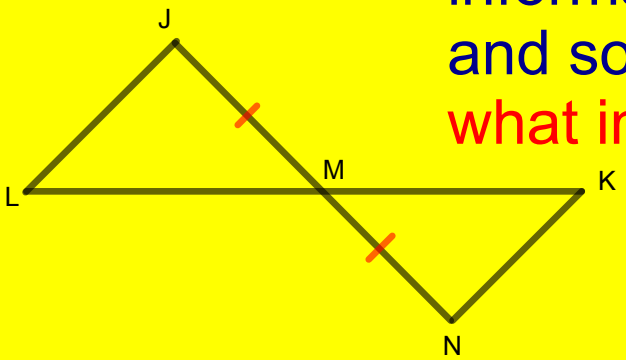
~ Give a reason for every statement

~ Use given information, definitions, postulates and theorems as reasons

~ List statements in order. If a statement relies on a previous statement, it must come later in the proof

~ End the proof with the statement you are trying to prove

Setting Up the Proof

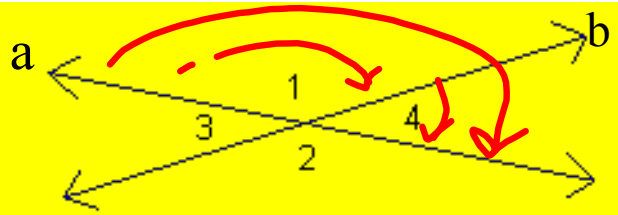


First, look carefully at the diagram. Lots of information is contained here, some visible and some hidden. Think about and discuss what information is contained in the diagram.

Set up the proof in two columns. Statements go on the left, reasons go on the right.

Given: Line a and line b are intersecting lines.

Prove: $m\angle 1 = m\angle 2$



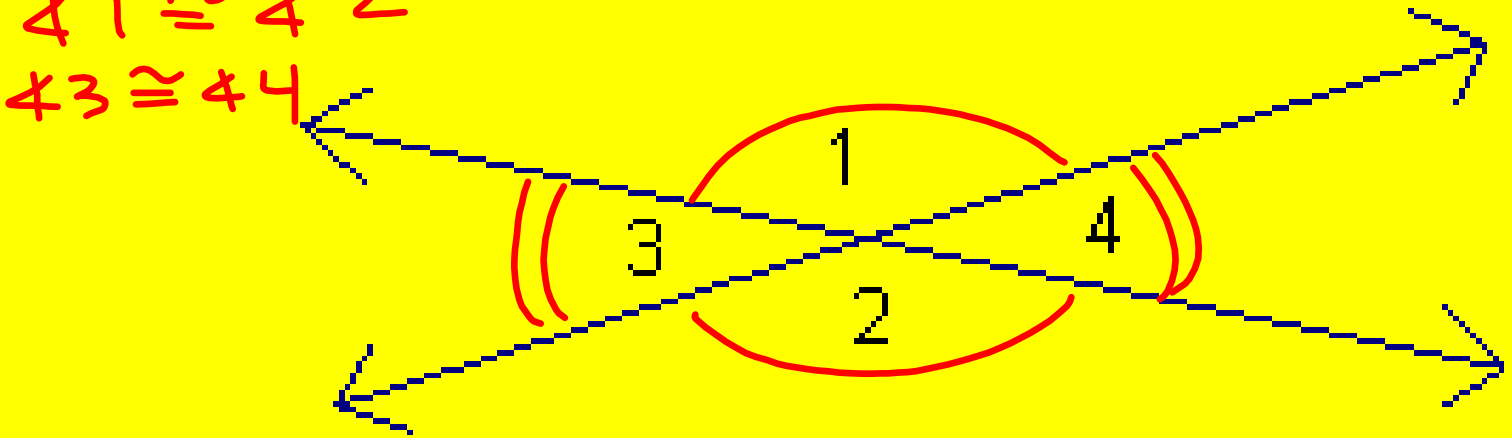
statement	reason
① Line a and line b are intersecting lines	① Given
② $m\angle 1 + m\angle 4 = 180$ $m\angle 4 + m\angle 2 = 180$	② ∠ add post
③ $m\angle 1 + m\angle 4 = m\angle 4 + m\angle 2$	③ Subst prop =
④ $m\angle 1 = m\angle 2$	④ Subtr. prop =

Theorem 2-1: Vertical Angles Theorem - Vert.$\angle$ are $\cong$

Vertical angles are congruent.

$\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$

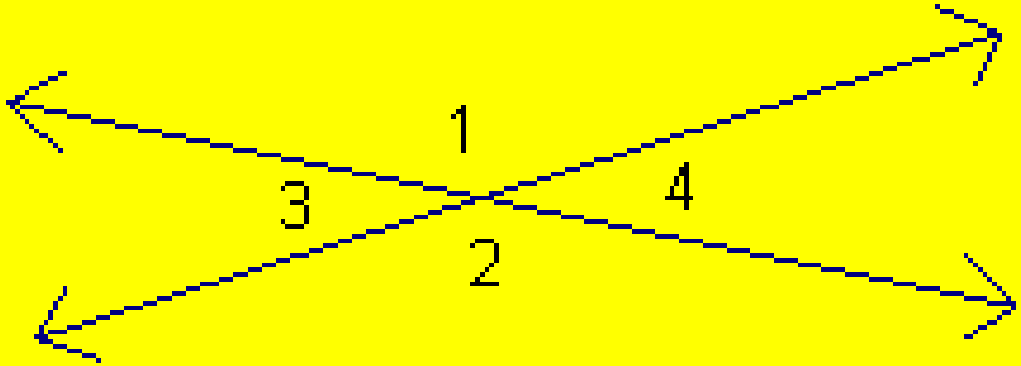


$\angle 1 \cong$ _____ and $\angle 3 \cong$ _____

Theorem 2-2: Congruent Supplements Theorem

Supple. of $\cong \angle$'s are $\cong$.

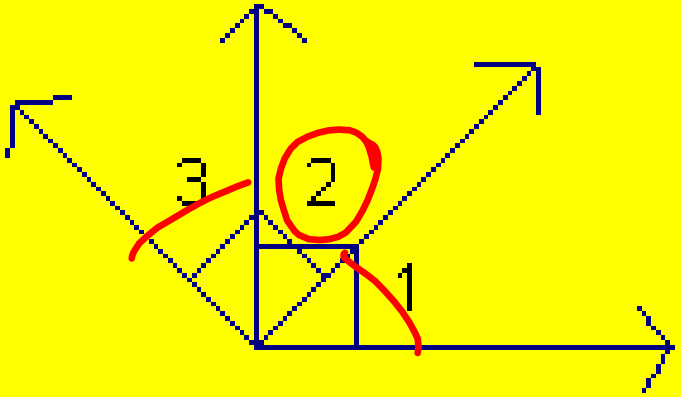
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.



Theorem 2-3: Congruent Complements Theorem

Compl. of $\cong$ $\angle$'s are $\cong$

If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.



$m\angle 3 + m\angle 2 = 90$

$m\angle 2 + m\angle 1 = 90$

$m\angle 3 + m\angle 2 = m\angle 2 + m\angle 1$

$m\angle 3 = m\angle 1$

Theorem 2-4: All right angles are congruent.

Theorem 2-5: If two angles are congruent and supplementary, then each is a right angle.

$m\angle 1 = m\angle 2$

$m\angle 1 + m\angle 2 = 180$

$90 \quad 90$

A **theorem**

1. Using the Vertical Angles are Congruent Theorem:

a) Find x: $4x - 101 = 2x + 3$

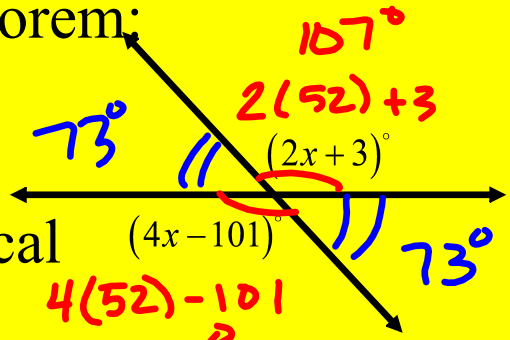
$-2x \quad -2x$

$2x - 101 = 3$

$+101 \quad +101$

$2x = 104$

$x = 52$



b) Find the measures of the labeled pair of vertical angles.

107° ; 107°

c) Find the measures of the other pair of vertical angles.

73° ; 73°

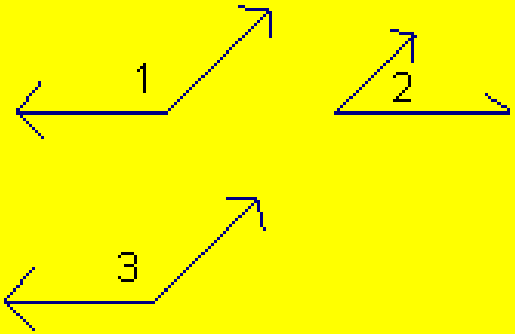
d) Check to see that adjacent angles are supplementary.

2. Write a two-column proof:

Given: $\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 2$ are supplementary

Prove: $m\angle 1 = m\angle 3$



Statement	Reasons
① ∠1 and ∠2 are supple. ∠3 and ∠2 are supple.	① Given
② m∠1 + m∠2 = 180 m∠2 + m∠3 = 180	② defn of supple ∠'s
③ m∠1 + m∠2 = m∠2 + m∠3	③ Subst prop =
④ m∠1 = m∠3	④ Subtr prop =

3. Write a two-column proof:

Given: WX = YZ

Prove: WY = XZ

Statement	Reason
① WX = YZ	① Given
② WX + XY = WY XY + YZ = XZ	② Seg. add post.
③ XY = XY	③ Reflexive prop =
④ WX + XY = YZ + XY	④ add prop =
⑤ WY = XZ	⑤ Subst prop =

4. a)Use the diagram and $m\angle ABX = 3x + 6$ and $m\angle RBC = 5x - 20$ to find x, $m\angle ABX$, and $m\angle XBC$.

"Vert $\angle$s $\cong$"

$3x + 6 = 5x - 20$

$-3x \quad -3x$

$6 = 2x - 20$

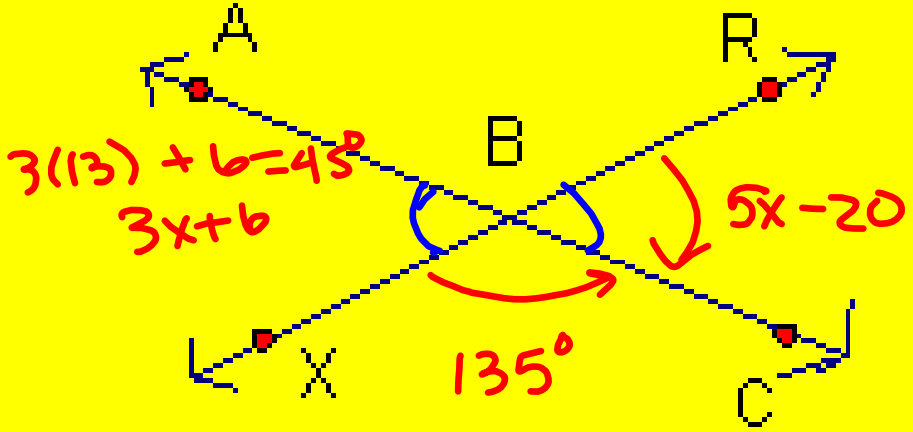
$+20 \quad +20$

$26 = 2x$

$x = 13$

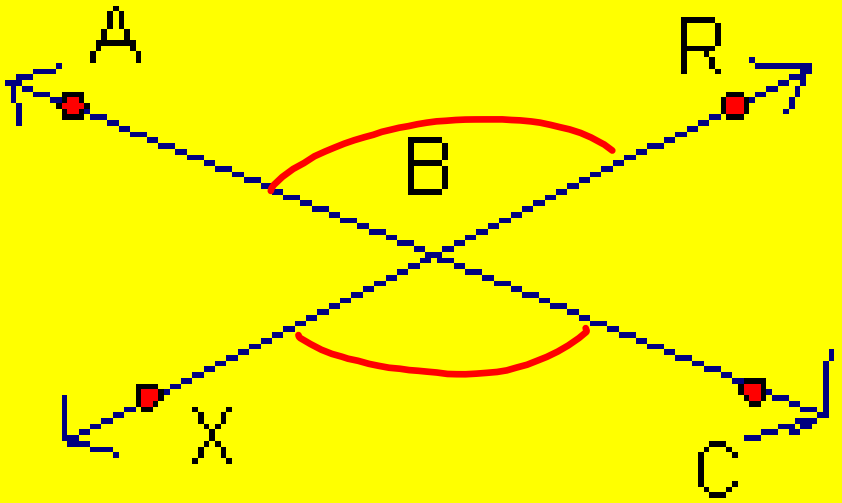
$m\angle ABX = 45^\circ$

$m\angle XBC = 135^\circ$



b) Without using the Vertical Angles are Congruent Theorem, what theorem can you use to prove that $\angle ABR \cong \angle XBC$.

Supple. of $\cong$'s are $\cong$



Find each angle measure given an angle with a measure 8 less than the measure of its complement.

angle = x

compl ∠ = 90 - x

$x = 90 - x - 8$

$x = 82 - x$

$+x \quad +x$

$2x = 82$

$x = 41$

angle = 41°

Compl. ∠ = 49°

Find each angle measure given that an angle with a measure three times the measure of its supplement.

angle = x = 135^0

Supple. $\angle$ = $180 - x = 180 - 135 = 45^0$

$x = 3(180 - x)$

$x = 540 - 3x$

$-3x \quad +3x$

$\frac{4x}{4} = \frac{540}{4}$

$x = 135$

Assignment

Day 1: Pgs 112-115 1,3,4,8,13,15, 16,17,23,25,33-39