Geometry

Ch. 4 Handout 4.5

Isosceles and Equilateral Triangles

1. Name the angle opposite $\overline{AB}$.

$\measuredangle C$

2. Name the angle opposite $\overline{BC}$.

$\measuredangle A$

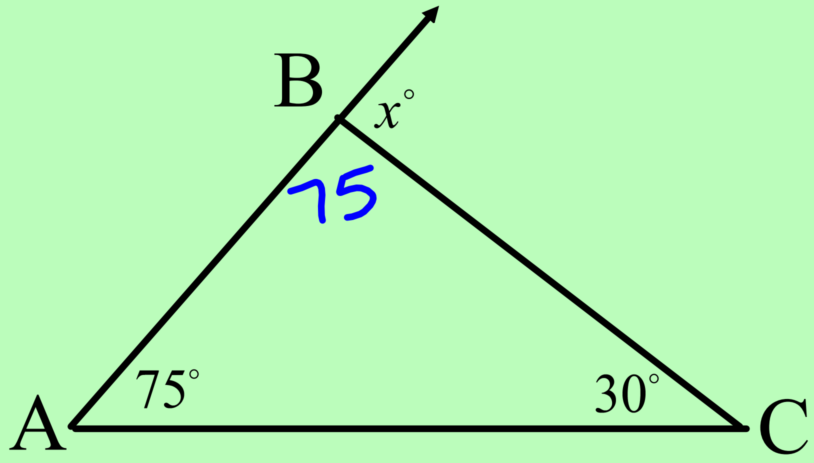
3. Name the side opposite $\angle A$.

$\overline{BC}$

4. Name the side opposite $\angle C$.

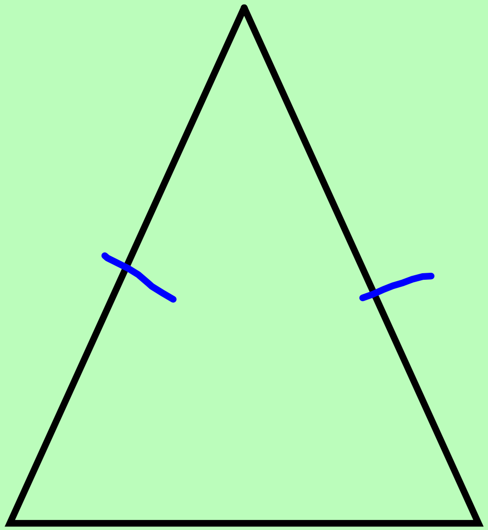
$\overline{AB}$

5. Find the value of x. 105



$$\begin{array}{r} 75 \\ 30 \\ \hline 105 \end{array} \qquad \begin{array}{r} 180 \\ -105 \\ \hline 75 \end{array}$$

Isosceles Triangle



Pull

two sides of the triangle congruent

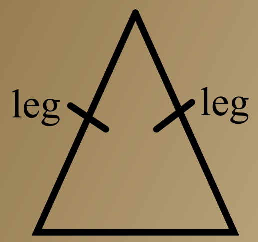
Isosceles triangles are common in the real world.

Parts of an Isosceles Triangle

legs

Pull

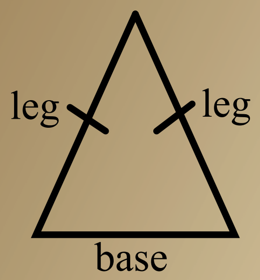
congruent sides of the triangle.



base

Pull

third side

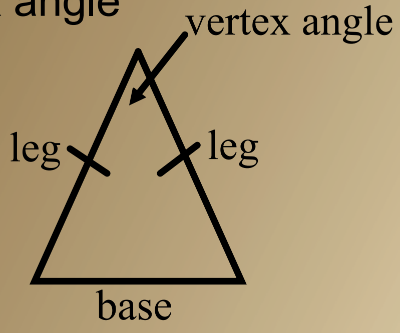


Parts of an Isosceles Triangle

vertex angle

Pull

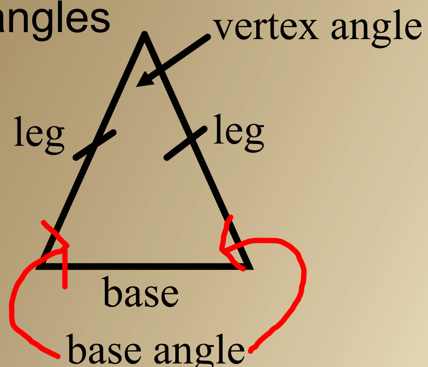
two congruent sides form the vertex angle

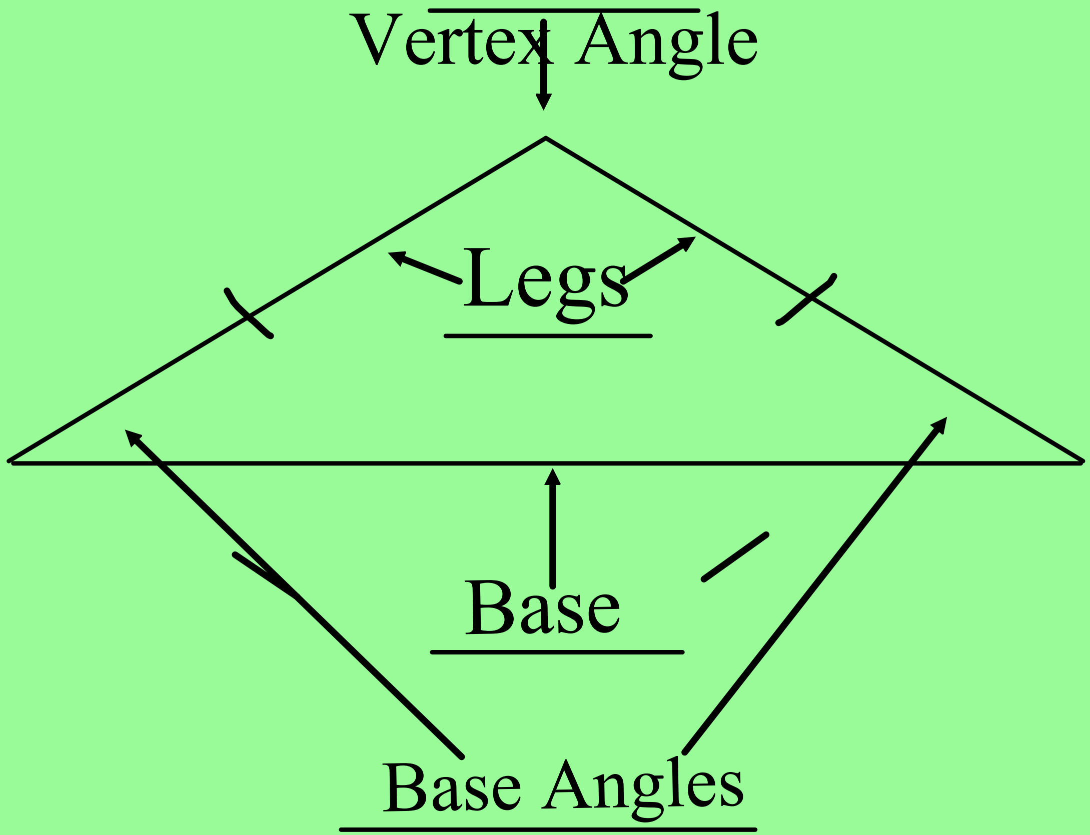


base angle

Pull

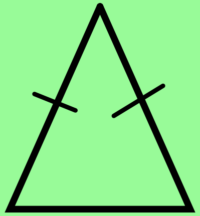
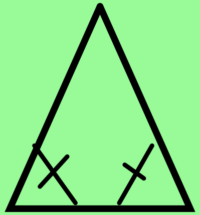
the other two angles

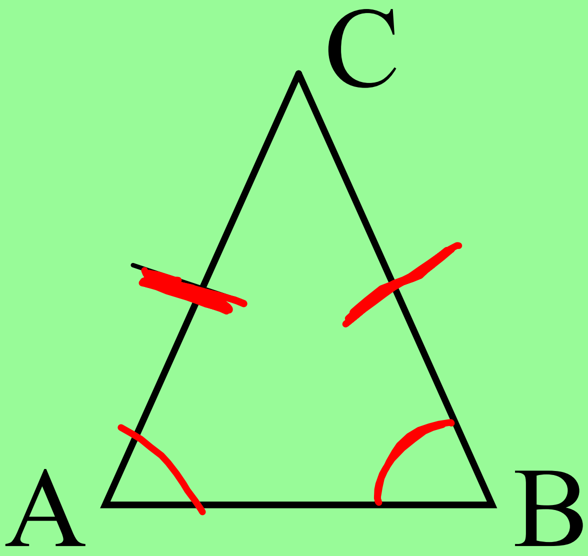



Vertex Angle
Legs
Base
Base Angles

Theorem 4.3: Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

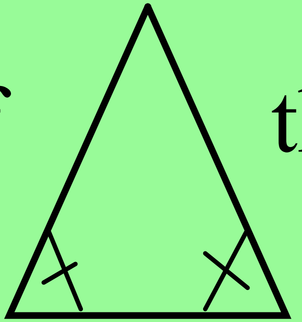
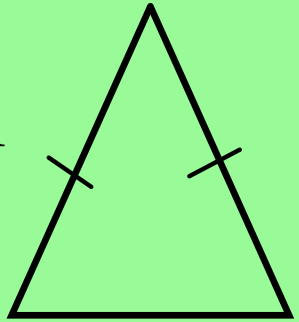
If  then .

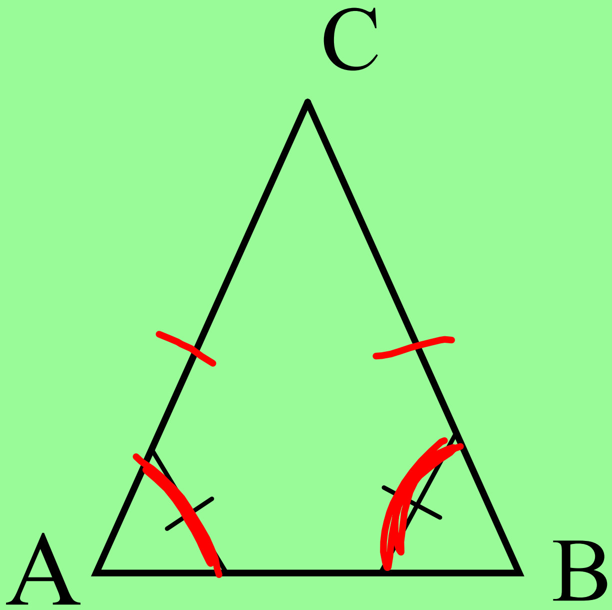


$\angle$ A ≅ $\angle$ B

Theorem 4.4: Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite the angle are congruent.

If  then  .

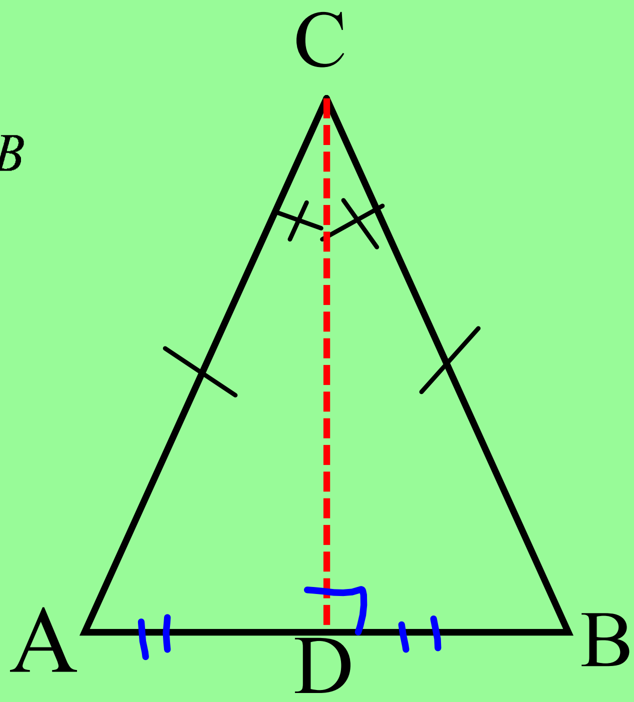


$\overline{AC}$ ≅ $\overline{CB}$

Theorem 4.5

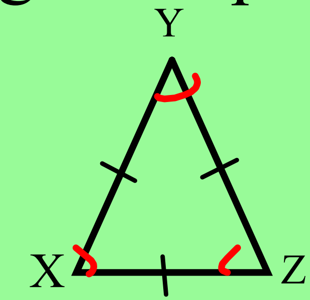
The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

If $\triangle ABC$ is isosceles, $\overline{CD}$ bisects $\angle ACB$ then $\overline{CD} \perp \overline{AB}$ and $\overline{AD} \cong \overline{DB}$.



Corollary to Theorem 4.3

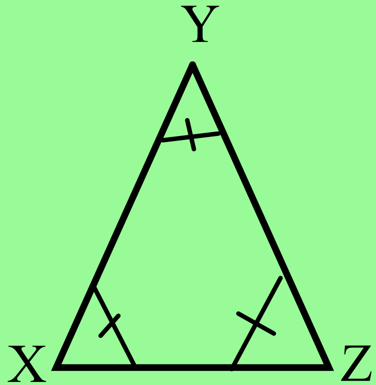
If a triangle is equilateral, then the triangle is equiangular.



$\angle$ X $\cong \angle$ Z $\cong \angle$ Y

Corollary to Theorem 4.4

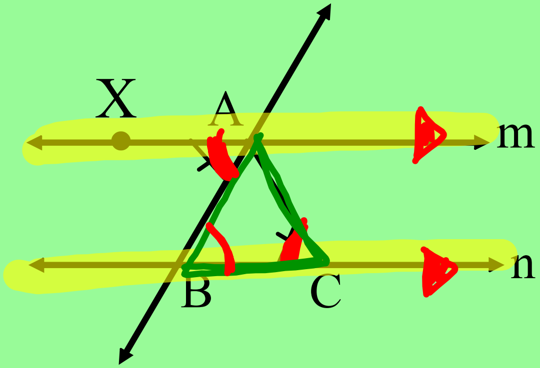
If a triangle is equiangular, then the triangle is equilateral.



$\overline{XY} \cong \overline{YZ} \cong \overline{XZ}$

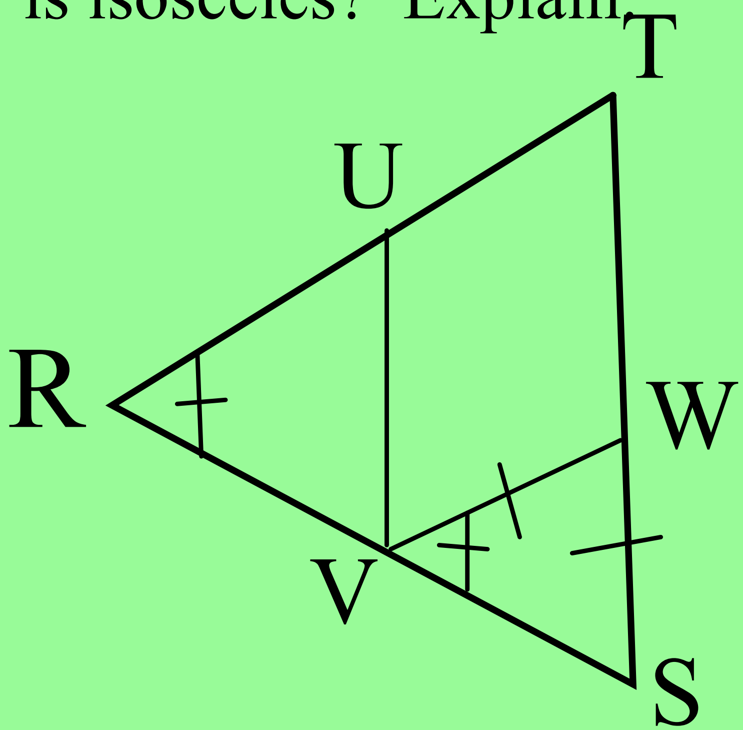
1. Given: $m \parallel n$, $\angle XAB \cong \angle C$

Prove: ΔABC is an isosceles triangle



Statements	Reasons
① $m \parallel n$; $\measuredangle XAB \cong \measuredangle C$	① Given
② $\measuredangle ABC \cong \measuredangle XAB$	② If $\parallel$ lines alt int $\measuredangle$'s $\cong$
③ $\measuredangle ABC \cong \measuredangle C$	③ trans. prop $\cong$
④ $\overline{AB} \cong \overline{AC}$	④ If △ then △
⑤ ΔABC is an isosceles △	⑤ defn of Isosceles △

2. Can you deduce that ΔRUV is isosceles? Explain.

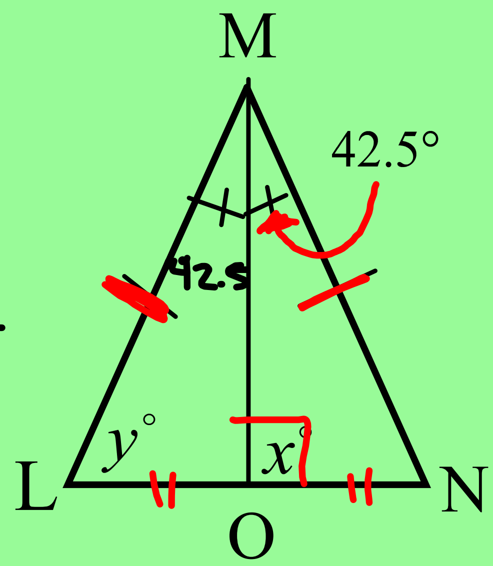


3. Suppose that $m\angle L = y$. Find the values of x and y.

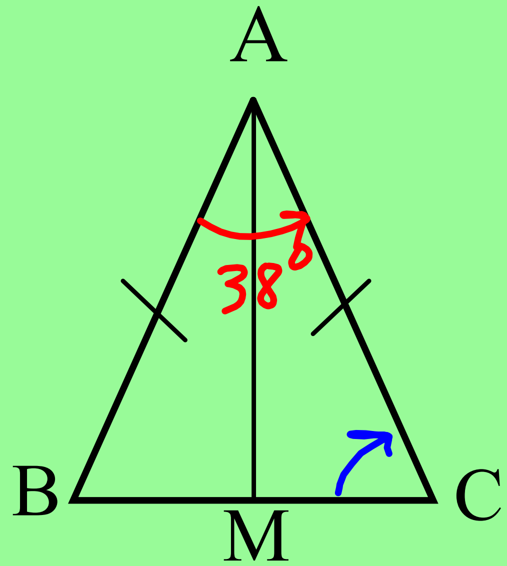
$x = 90^\circ$

$y = 47.5^\circ$

$$\begin{array}{r} 90 \\ -\ 42.5 \\ \hline \end{array}$$

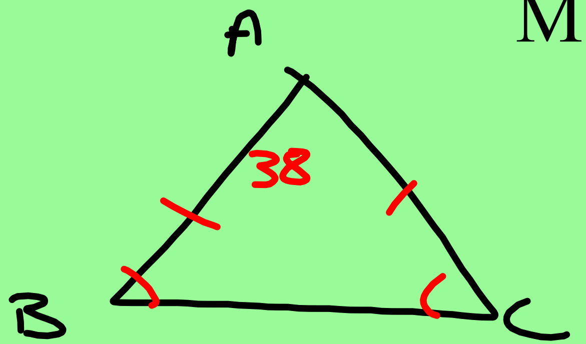


4. If $m\angle BAC = 38$, find $m\angle C$.



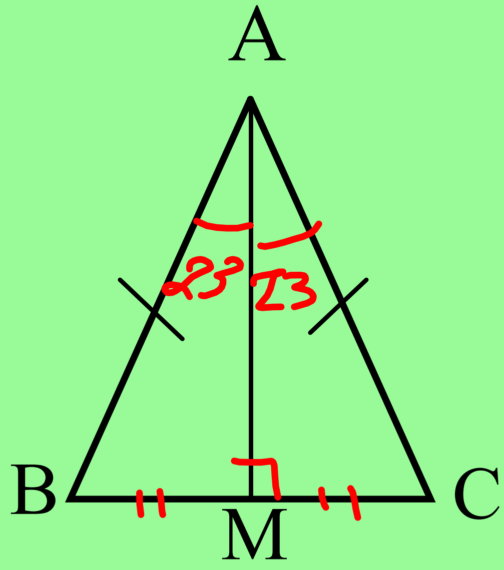
$\boxed{m\angle C = 71^\circ}$

$$\begin{array}{r} 180 \\ -\ 38 \\ \hline 142 \end{array} \qquad 142 \div 2 = 71$$

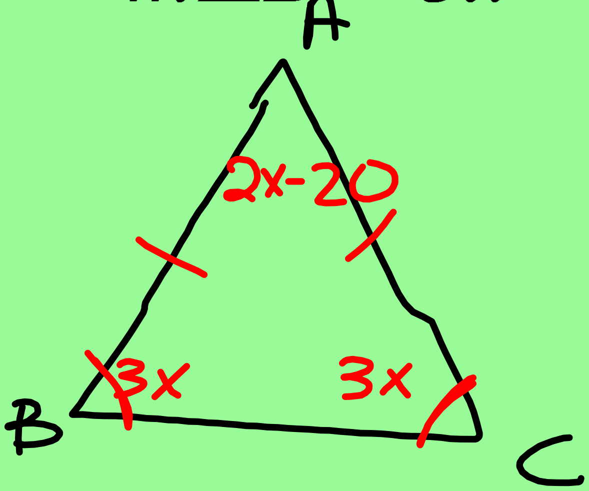


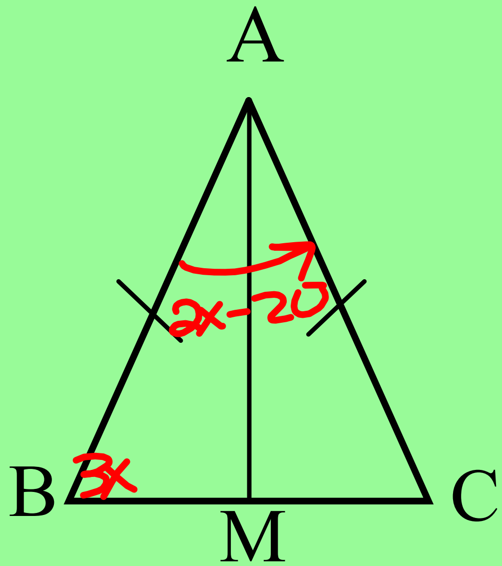
5. If $m\angle BAM = m\angle CAM = 23$, find $m\angle BMA$.

$$\boxed{m\angle BMA = 90^\circ}$$



6. If $m\angle B = 3x$ and $m\angle BAC = 2x - 20$, find x.





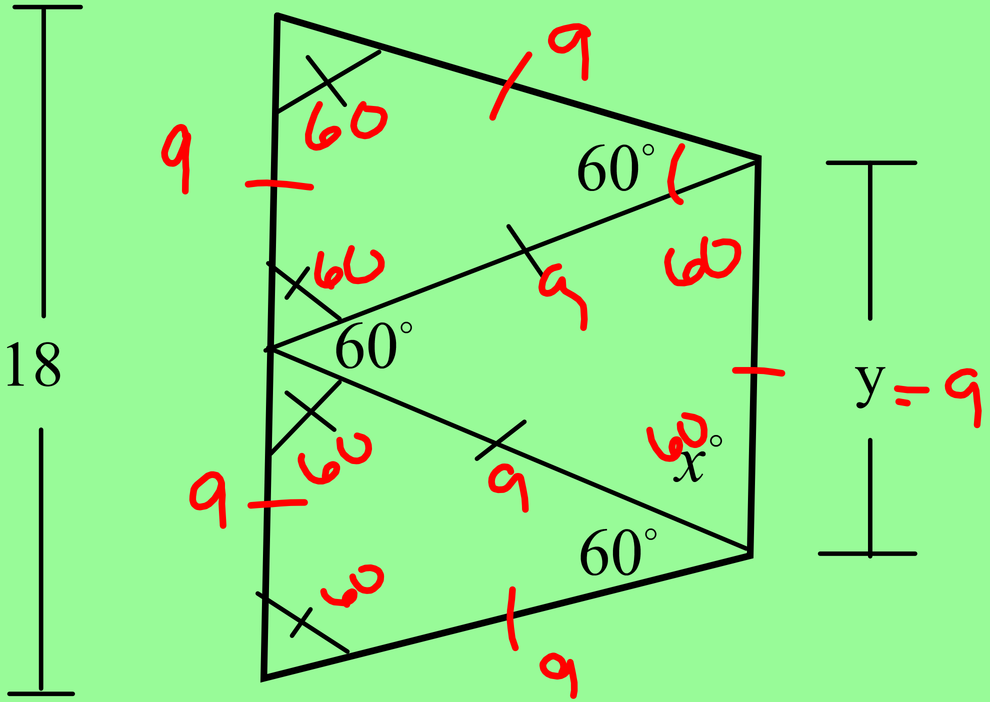
$3x + 3x + 2x - 20 = 180$

$8x - 20 = 180$
$+20 \quad +20$

$8x = 200$

$\boxed{x = 25}$

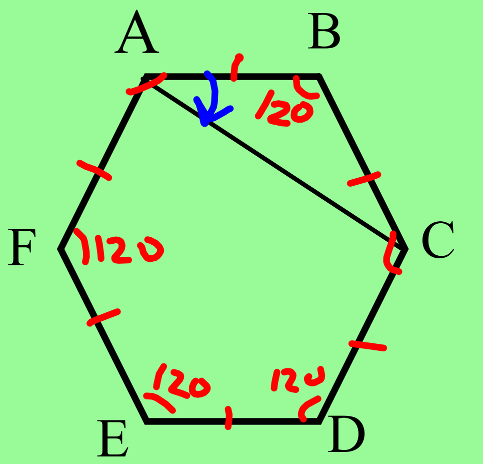
4. Find the values of x and y.



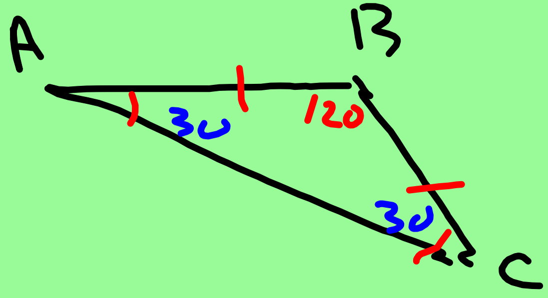
$x = 60$

$y = 9$

5. ABCDEF is a regular hexagon. Find $m\angle BAC$.



$$\frac{(6-2)(180) = 720}{6} = 120$$



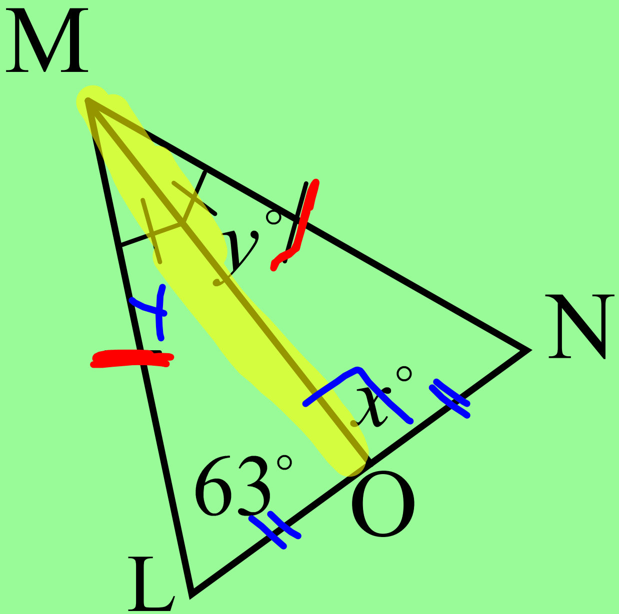
$$\begin{array}{r} 180 \\ -\,120 \\ \hline 60 \end{array} \qquad 2\overline{)\,60}^{\,30}$$

$$\boxed{m\angle BAC = 30^\circ}$$

6. Suppose $m\angle L = 63^\circ$. Find the values of x and y.

$\boxed{\begin{array}{l} x = 90^\circ \\ y = 27^\circ \end{array}}$

$$\begin{array}{r} 90 \\ -\,63 \\ \hline 27 \end{array}$$



Assignment:

Pgs 230-233 1-13, 19-22, 30-32, 36