Geometry

Ch. 5 Handout 5.2

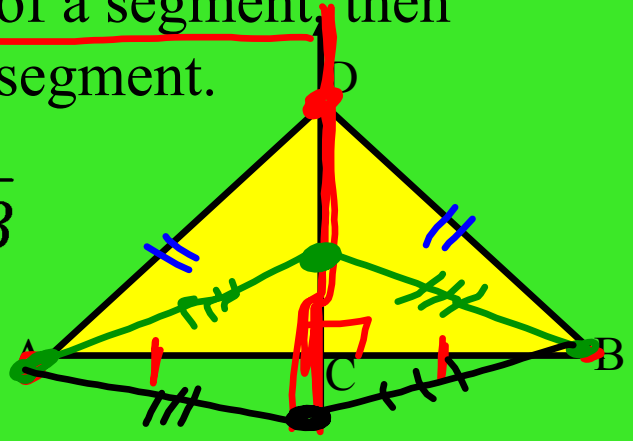
Bisectors in Triangles

Theorem 5.2: Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

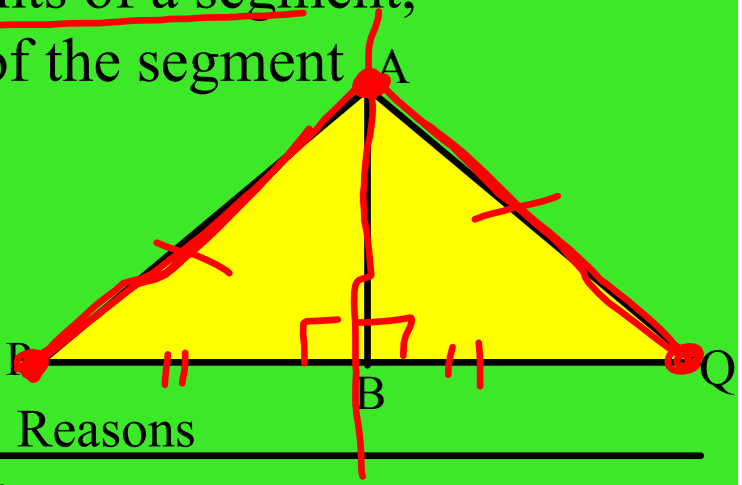
Given: $\overleftrightarrow{CD} \perp \overline{AB}$, $\overleftrightarrow{CD}$ *bisects* $\overline{AB}$

Prove: $DA = DB$



Theorem 5.3: Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.



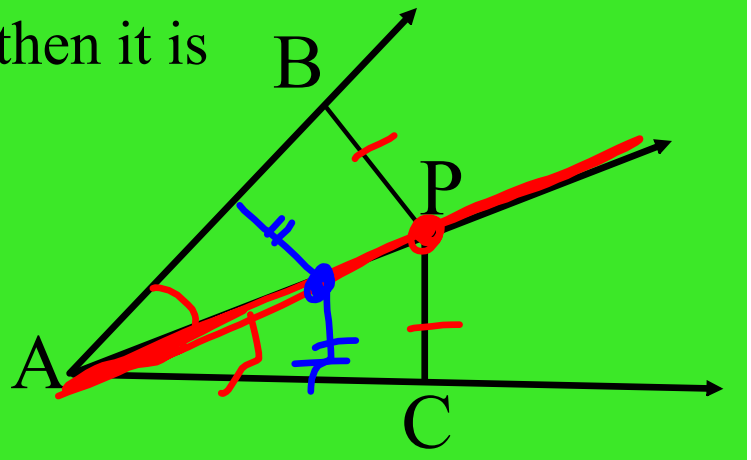
Given: AP = AQ with $\overline{AB} \perp \overline{PQ}$ at B

Prove: $\overline{AB}$ is the perpendicular bisector of $\overline{PQ}$

Statement	Reasons
① AP = AQ; $\overline{AB} \perp \overline{PQ}$ at B	① Given
② m∠ABP = 90; m∠ABQ = 90	② defn of ⊥ lines
③ △ABP and △ABQ are rt △s	③ defn of Rt △'s
④ △ABP ≅ △ABQ	④ HL thm
⑤ $\overline{PB} \cong \overline{QB}$	⑤ CPCTC
⑥ $\overline{AB}$ is the ⊥ bisector of $\overline{PQ}$	⑥ defn of ⊥ bisector

Theorem 5.4: Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.



Given: $\overline{PB} \perp \overrightarrow{AB}, \overline{PC} \perp \overrightarrow{AC}$,
$\overrightarrow{AP}$ bisects $\angle BAC$

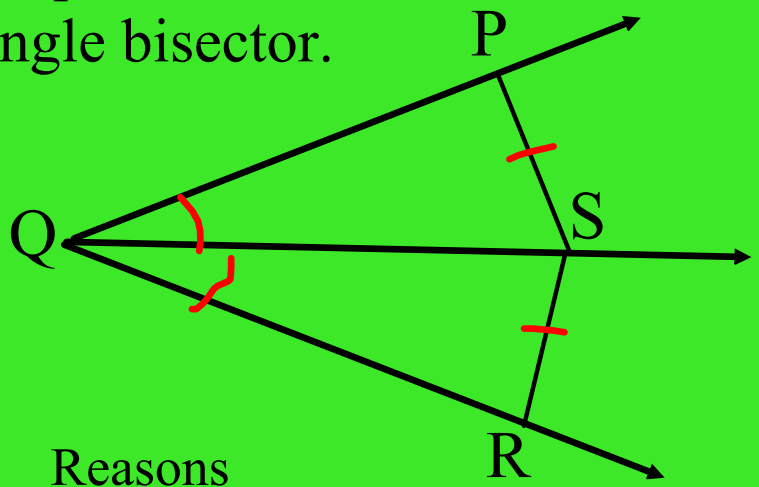
Prove: PB = PC

Theorem 5.5: Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

Given: $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$
SP = SR

Prove: $\overrightarrow{QS}$ bisects $\angle PQR$



Statement	Reasons
① $\overline{SP} \perp \overline{QP}$; $\overline{SR} \perp \overline{QR}$, SP = SR	① Given
② m∠SPQ = 90, m∠SRQ = 90	② defn of ⊥ lines
③ m∠SPQ = m∠SRQ	③ Subst
④ $\overline{SQ} \cong \overline{SQ}$	④ Reflexive prop ≅
⑤ $\triangle QPS \cong \triangle QRS$	⑤ HL thm
⑥ ∠PQS ≅ ∠RQS	⑥ CPCTC
⑦ $\overrightarrow{QS}$ bisects ∠PQR	⑦ defn of ∠ bisector

The distance from point to a line is

Pull

the length of the perpendicular segment from the point to the line

1. Find x, FB, and FD in the diagram at the right.

$2x+5 = 7x-37$

$-2x \quad -2x$

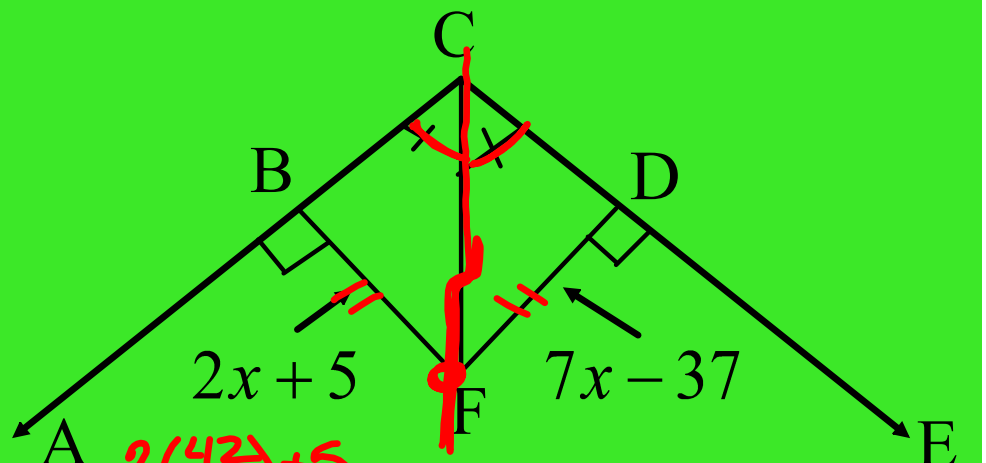
$5 = 5x-37$

$+37 \quad +37$

$42 = 5x$

$x = \frac{42}{5}$

$BF = \frac{109}{5} = FD$



$2\left(\frac{42}{5}\right)+5$

$\frac{84}{5}+\frac{25}{5}=\frac{109}{5}$

If a point lies on the angle bisector, than the point is equidistant from the sides of the angle.

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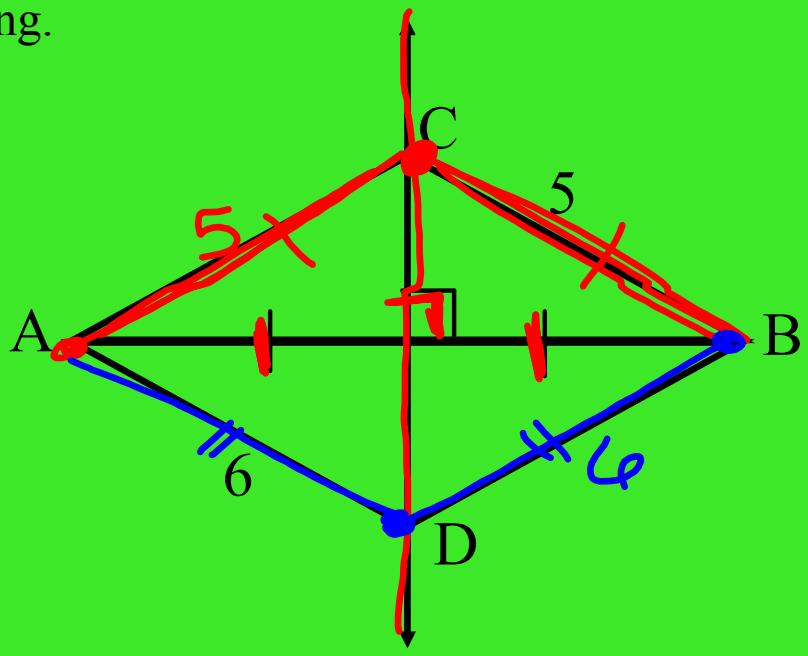
2. Use the information given in the diagram. $\overline{CD}$ is the perpendicular bisector of AB. Find CA and DB. Explain your reasoning.

CA = 5

DB = 6



If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

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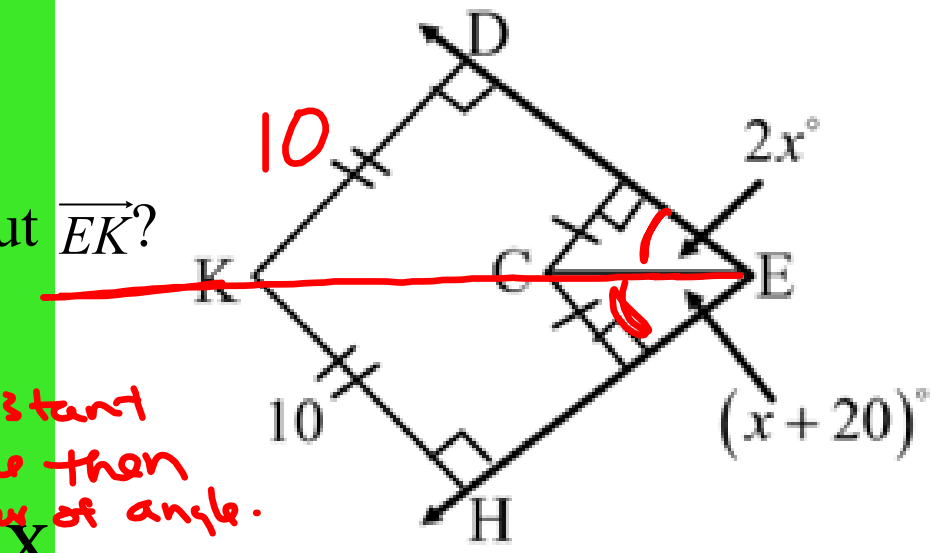
3. a. According to the diagram, how far is K from EH? From ED?



b. What can you conclude about $\overline{EK}$?

4 bisector

If a point is equidistant from the sides of an angle then point is on the bisector of angle.

c. Find the value of x.

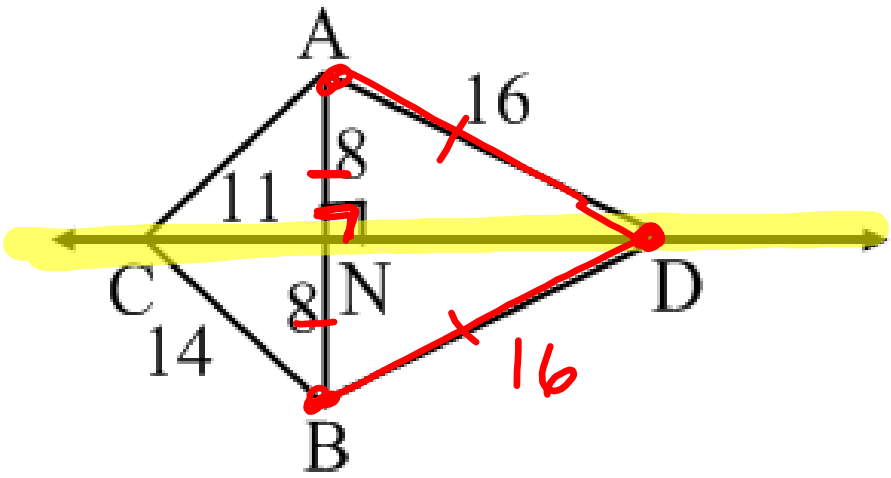
$2x = x + 20$

$x = 20$

d. Find $m\angle DEH$.

Use the figure at the right for exercises 4-5.

4. Find BD= 16



5. Complete the statement: C is equidistant from A and B .

Use the figure at the right for exercises 6-8.

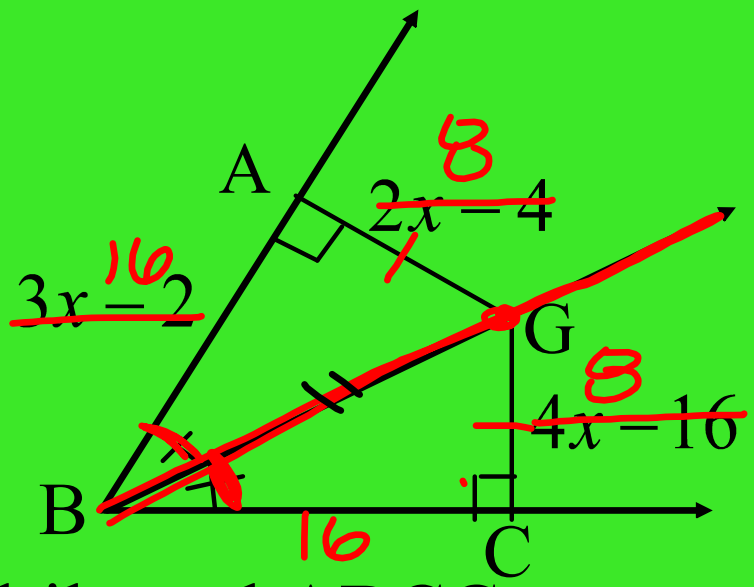
6. Find the value of x.

$2x - 4 = 4x - 16$

$-2x + 16 \quad -2x + 16$

$12 = 2x$ $\boxed{x = 6}$

7. Find CG. = 8



8. Find the perimeter of quadrilateral ABCG.

$16 + 16 + 8 + 8 = 48$

Assignment:

pg 267 1-4, 6-16, 18-25, 28-30, 46, 47