Algebra 2

Ch. 6 Handout 6.2

Polynomials and Linear Factors

Write the polynomial in standard form.

$$(x-1)(x+3)(x+4)$$

$(x^2+4x+3x+12)$

$(x-1)(x^2+7x+12)$

$x^3+7x^2+12x-x^2-7x-12$

$$x^3+6x^2+5x-12$$

2. Write the polynomial in standard form.

$$x(x-3)^2$$

$x[(x-3)(x-3)]$

$(x^2-3x-3x+9)$

$x(x^2-6x+9)$

$$x^3-6x^2+9x$$

3. Write the polynomial in factored form. Check by multiplication.

$$3x^3 - 18x^2 + 24x$$

$3x$ $3x$ $3x$

$3x(x^2 - 6x + 8)$

$3x(x-4)(x-2)$

4. Write each polynomial in factored form. Check by multiplication.

$$\frac{3x^4}{3x^2} - \frac{3x^3}{3x^2} - \frac{36x^2}{3x^2}$$

$$3x^2(x^2 - x - 12)$$

$$\boxed{3x(x-4)(x+3)}$$

A multiple zero is

Pull

a zero of a linear factor that is repeated in the factored form of the polynomial.

The multiplicity of a zero of a polynomial function is

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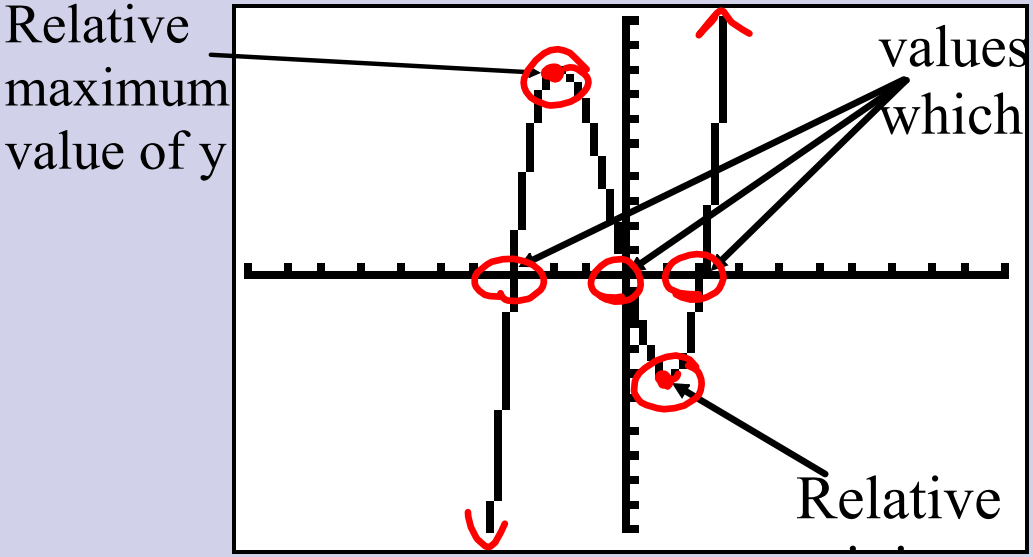
the number of times the related linear factor is repeated in the factored form of the polynomial


Relative maximum value of y
x-intercepts: values of x for which y = 0
Relative minimum value of y

Find the zeros of each function. State the multiplicity of multiple zeros.

$$y = x^2(x-1)^3(x+3)$$

$0 = (x)(x)(x-1)(x-1)(x-1)(x+3)$

$x=0 \quad x=0 \quad x-1=0 \quad x-1=0 \quad x-1=0 \quad x+3=0$

$x=1 \quad x=1 \quad x=1 \quad x=-3$

Zeros: $x=0$ mult. 2
$x=1$ mult. 3
$x=-3$

Find the zeros of each function. State the multiplicity of multiple zeros.

$$y = \left(x + \frac{1}{2}\right)(x-4)^3$$

$0 = (x+\frac{1}{2})(x-4)(x-4)(x-4)$

$x+\frac{1}{2}=0$ $x-4=0$ $x-4=0$ $x-4=0$

$x=-\frac{1}{2}$ $x=4$ $x=4$ $x=4$

Zeros: $x=-\frac{1}{2}$, $x=4$ mult. 3

Find the zeros of each function. State the multiplicity of multiple zeros.

$$y = x^3 - 2x^2 - 48x$$

$0 = x(x^2 - 2x - 48)$

$0 = x(x - 8)(x + 6)$

$x = 0 \quad x - 8 = 0 \quad x + 6 = 0$

$x = 8 \quad x = -6$

Zeros: $x = 0, x = -6, x = 8$

Find the zeros of each function. State the multiplicity of multiple zeros.

$$f(x) = x^3 - 81x$$

$y = x^3 - 81x$

$0 = x(x^2 - 81)$

$0 = x(x-9)(x+9)$

$x = 0 \quad x - 9 = 0 \quad x + 9 = 0$

$x = 9 \quad x = -9$

Zeros: $x = 0, x = 9, x = -9$

Find the zeros of each function. State the multiplicity of multiple zeros.

$$y = x^4 + 3x^3 - x^2 - 3x$$

$0 = x(x^3 + 3x^2 - x - 3)$

$0 = x[x^3 + 3x^2 + -x + -3]$

$0 = x[(x^3 + 3x^2) + (-x + -3)]$

$0 = x[x^2(x+3) + -1(x+3)]$

$0 = x(x+3)(x^2 - 1)$

$0 = x(x+3)(x-1)(x+1)$

$x = 0 \quad x+3 = 0 \quad x-1 = 0 \quad x+1 = 0$

Zeros: $x = 0, x = -3, x = 1, x = -1$

Factor Theorem: The x - a is a linear factor of a polynomial if and if the only if the value a is a zero of the related polynomial function.

Write a polynomial in standard form with zeros at 2, -3, and 0.

$x = 0 \quad x = 2 \quad x = -3$

$-2 \quad -2 \quad +3 \quad +3$

$x = 0 \quad x - 2 = 0 \quad x + 3 = 0$

$x[(x-2)(x+3)] = 0$

$x^2 + 3x - 2x - 6$

$x(x^2 + x - 6) = y$

$x^3 + x^2 - 6x = y$

Write a polynomial in standard form with zeros at 4, -1, and -1

$x = 4$ $\quad x = -1$ $\quad x = -1$

$-4 \quad -4 \quad +1 \quad +1 \quad +1 \quad +1$

$x - 4 = 0 \quad x + 1 = 0 \quad x + 1 = 0$

$(x-4)(x+1)(x+1) = 0$

$x^2 + x + x + 1$

$(x-4)(x^2+2x+1) = y$

$x^3 + 2x^2 + x - 4x^2 - 8x - 4 = y$

$$\boxed{x^3 - 2x^2 - 7x - 4 = y}$$

Write a polynomial in standard form: 2 multiplicity 3

$x = 2 \quad x = 2 \quad x = 2$

$x - 2 = 0 \quad x - 2 = 0 \quad x - 2 = 0$

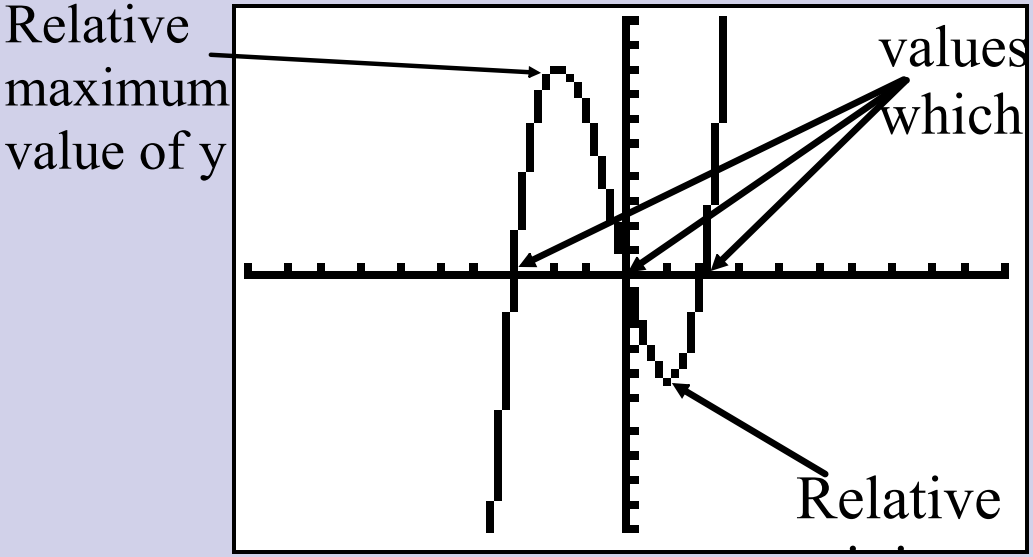
$(x-2)(x-2)(x-2) = 0$

$x^2 - 2x - 2x + 4$

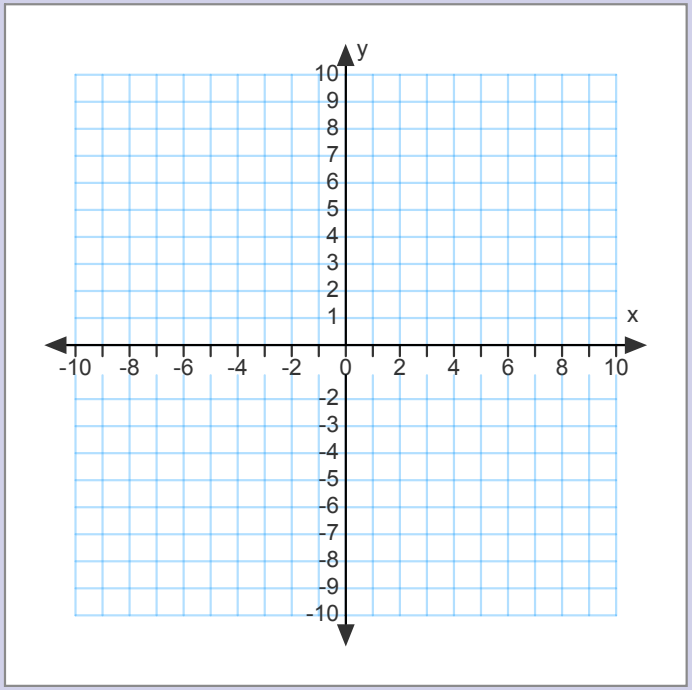
$(x-2)(x^2 - 4x + 4) = y$

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 = y$

$$\boxed{x^3 - 6x^2 + 12x - 8 = y}$$

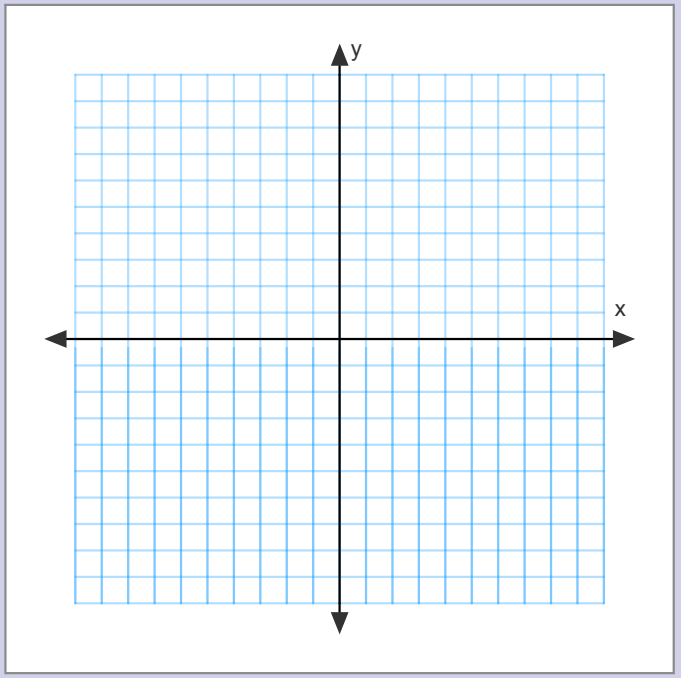

Relative maximum value of y
x-intercepts: values of x for which y = 0
Relative minimum value of y

Find the relative maximum, relative minimum, and zeros of $y = (x+1)(x-1)(x+3)$. Graph the function.



12. Find the relative maximum, relative minimum, and zeros of the polynomial function.

$$y = x^3 - 2x^2 - 15x$$

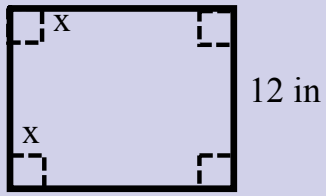


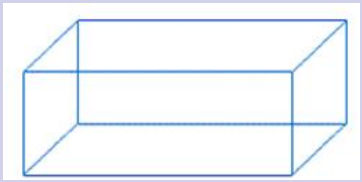
A metalworker wants to make an open box from a sheet of metal, by cutting equal squares from each corner as shown.

a) Write an expression for the length, width, and height of the open box.

b) Use your answer from part (a) to write a function for the volume.

c) Graph the function. Find the maximum volume that can be contained by the box and the size of the square cut that produces this volume.





3. Another airline has different carry-on luggage regulations. The sum of the length, width, and depth may not exceed 50 in.

a) Assume that the sum of the length, width, and depth is 50 in. and the length is 10 in. greater than the depth. Graph the function relating the volume V to depth x. Find the x-intercepts. What do they represent?

b) Describe a realistic domain for V(x).

c) What is the maximum possible volume of the box? What are the corresponding dimensions of the box?

Pull

Pull

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

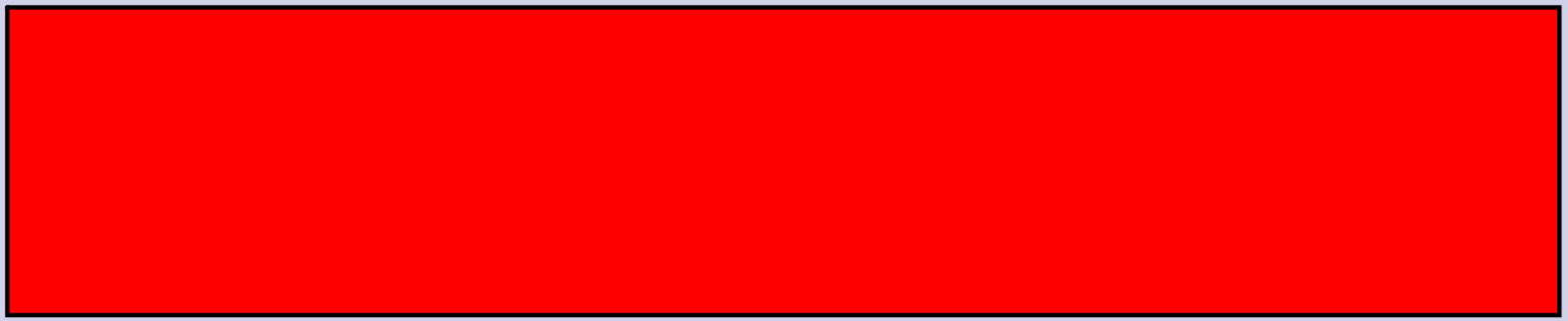
$x + 4 = 0 \quad x - 1 = 0$

$x = -4 \quad x = 1$

Equivalent Statements about Polynomials

1. -4 or 1 are solutions of $x^2 + 3x - 4 = 0$
2. -4 or 1 are x-intercepts of the graph of $y = x^2 + 3x - 4$
3. -4 or 1 are zeros of $y = x^2 + 3x - 4$
4. $(x-1)$ and $(x+4)$ are factors of $x^2 + 3x - 4$

Assignments:



Day 3: pgs 317 (13, 38, 39, 40, 48, 50)