Algebra 2

Ch. 6 Handout 6.7

Permutations and Combinations

n Factorial

For any positive integer n $n! = n(n-1)(n-2)\cdot\ldots\cdot 3\cdot 2\cdot 1$

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Factorial --

$0! = 1$

$1! = 1$

$2! = 2\cdot 1 = 2$

$3! = 3\cdot 2\cdot 1 = 6$

$4! = 4\cdot 3\cdot 2\cdot 1 = 24$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$

$7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 5040$

$8! = 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 40320$

Simplify:

1) $9! = 362,880$

$= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

2) $\dfrac{14!}{10!4!} = 1001$

$= \dfrac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{10! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= \dfrac{7 \cdot 13 \cdot 11}{1}$

$= 1001$

Permutations

1. is an arrangement of items in a particular order Pull
2. order matters Pull
3. details matter/position matters Pull

Combinations

1. a selection in which order does not matter

Pull

2. doesn't change the look

Pull

3. order does not matter -- you can mix it up and looks the same

Pull

4.

Pull

1. State whether each one is Combination or Permutation.

a) Picking a team of 3 people from a group of 10.

Pull
Combination
order does not matter

b) Picking a president, vice-president, and a water boy from a group of 12

Pull
Permutation
order matters

c) 3 desserts from a menu of 10.

Pull
Combination
order does not matter

d) Listing your 3 favorite desserts, in order, from a menu of 10 items.

Pull
Permutation
order matters

Number of Permutations

The number of permutations of n items of a set arranged r items at a time is ${}_nP_r$.

$$ {}_nP_r = \frac{n!}{(n-r)!} \quad \text{for} \quad 0 \le r \le n $$

Ex: ${}_8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \cdot 7 \cdot 6!}{6!}$

${}_8P_2 = 56$ ways

Number of Combinations

The number of combinations of n items of a set chosen r items at a time is ${}_nC_r$.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \le r \le n $$

Ex: ${}_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1 \cdot 4!}$

${}_7C_3 = 35$ ways

In how many ways can 8 people line up from left to right for a group photo?

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= \boxed{40{,}320 \text{ ways}}$

How many 4-letter codes can be made ifno letter can be used twice?

Permutation or Combination? **Permutation**

$$_{N}P_{R} = {}_{26}P_{4} = \frac{26!}{(26-4)!} = \frac{26!}{22!}$$

$$= \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot \cancel{22!}}{\cancel{22!}}$$

$$= 358{,}800 \text{ ways}$$

A disc jockey wants to select 5 songsfrom a new CD that contains 12 songs. How many 5-song selectionsare possible?

Permutation or Combination? **Combination**

$$_NC_R = {}_{12}C_5 = \frac{12!}{5!(12-5)!}$$

$$= \frac{12!}{5!\,7!} = \frac{12 \cdot 11 \cdot \overset{2}{10} \cdot 9 \cdot \overset{4}{8} \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!}$$

$$= 11 \cdot 2 \cdot 9 \cdot 4$$

$$= 792 \text{ ways}$$

Of the 12 songs, in how many ways can the disc jockey select seven songs? Twelve songs?

Permutation or Combination? **Combination**

$$_{12}C_7 = \frac{12!}{7!(12-7)!} = \frac{12!}{7!5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 11 \cdot 9 \cdot 8$$

$$= 792 \text{ ways}$$

$$_{12}C_{12} = \frac{12!}{12!(12-12)!} = \frac{12!}{12! \cdot 1!} = \frac{1}{1} = 1 \text{ way}$$

A *combination* is when things are selected, and a *permutation* is when things are arranged.

Review

5. In how many ways can you arrange six trophies on a shelf?

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ 720 ways

6. How many 3-letter codes can be made if no letter can be used twice?

$$_{26}P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!} = \frac{26 \cdot 25 \cdot 24 \cdot \cancel{23!}}{\cancel{23!}}$$

$$= 15,600 \text{ ways}$$

Permutation or Combination? **Permutation**

The ski club with ten members is to choose three officers captain, co-captain, and secretary, how many ways can those offices be filled?

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$$

$$= 720 \text{ ways}$$

Permutation or Combination?

There are 12 standbys who hope to get on your flight to Hawaii, but only 6 seats are available on the plane. How many different ways can the 6 people be selected?

$$_{12}C_{6} = \frac{12!}{6!(12-6)!} = \frac{12!}{6!\,6!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 11 \cdot 2 \cdot 3 \cdot 2 \cdot 7$$

$$= 924 \text{ ways}$$

Assignment:

6.7 pgs 348-350 2-16 evens,18,19,22-28 evens,
29,30,43,45,46- 49,50,52,56,58