Algebra 2

Ch. 7 Handout 7.4

Rational Exponents

Rational Exponents

If the nth root of a is a real number and m is an integer, then

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$. If m is negative, $a \neq 0$.

Radical Form → Exponential Form

$\sqrt[n]{b^m} \longrightarrow b^{\frac{m}{n}}$

Exponential Form → Radical Form

$b^{\frac{m}{n}} \longrightarrow \sqrt[n]{b^m} \rightarrow \left(\sqrt[n]{b}\right)^m$

Radical Form	Exponential Form
$\sqrt[4]{x^3}$	$x^{\frac{3}{4}}$
$\sqrt[5]{2^3}$	$2^{\frac{3}{5}}$
$\sqrt[6]{x^2}$	$x^{\frac{2}{6}} = x^{\frac{1}{3}}$
$\sqrt[7]{x^4}$	$x^{\frac{4}{7}}$

Properties of Rational Expressions

Property	Example
$a^m \cdot a^n = a^{m+n}$	$8^{\frac{1}{3}} \cdot 8^{\frac{2}{3}} = 8^{\frac{1}{3}+\frac{2}{3}} = 8^{\frac{3}{3}} = 8^1 = \boxed{8}$
$\left(a^m\right)^n = a^{mn}$	$\left(5^{\frac{1}{2}}\right)^4 = 5^{\frac{1}{2}\cdot 4} = 5^2 = \boxed{25}$
$(ab)^m = a^m b^m$	$(4\cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} 5^{\frac{1}{2}} = \sqrt{4}\sqrt{5} = \boxed{2\sqrt{5}}$ OR $(20)^{\frac{1}{2}} = \sqrt{20} = \sqrt{4\cdot 5} = 2\sqrt{5}$
$\left(a^{-m}\right) = \dfrac{1}{a^m}$	$9^{-\frac{1}{2}} = \dfrac{1}{9^{1/2}} = \dfrac{1}{\sqrt{9}} = \boxed{\dfrac{1}{3}}$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{2^{\frac{3}{2}}}{2^{\frac{1}{2}}} = 2^{\frac{3}{2}-\frac{1}{2}} = 2^{\frac{2}{2}} = 2^1 = \boxed{2}$
$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{5}{27}\right)^{\frac{1}{3}} = \dfrac{5^{\frac{1}{3}}}{27^{1/3}} = \dfrac{\sqrt[3]{5}}{\sqrt[3]{27}} = \boxed{\dfrac{\sqrt[3]{5}}{3}}$

Another way to write a radical expression is to use a rational exponents

Like the radical form, the exponent form always indicates the principal root.

A rational exponent may have a numerator other than 1.

All of the properties of integer exponents also apply to rational exponents.

You can simplify a number with a rational exponent by using the properties of exponents or by converting the expression to a radical expression.

To write an expression with rational exponents in simplest form write every exponent as a positive number.

1. Simplify each expression.

a) $64^{\frac{1}{3}} = \sqrt[3]{64^1} = \boxed{4}$

b) $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}$

$7^{\frac{1}{2}+\frac{1}{2}} = 7^{\frac{2}{2}} = 7^1 = \boxed{7}$

c) $5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}} = \sqrt[3]{5} \cdot \sqrt[3]{25} = \sqrt[3]{125} = \boxed{5}$

Simplify:

$\sqrt{x} \cdot \sqrt[3]{x}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$

$x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{3}{6}+\frac{2}{6}} = \boxed{x^{\frac{5}{6}}}$

$-64^{\frac{5}{6}}$

$-\left(\sqrt[6]{64}\right)^5$

$-(2)^5$

$\boxed{-32}$

2. Write the exponential expression in radical form.

a) $x^{\frac{2}{7}} = \boxed{\sqrt[7]{x^2}}$

b) $y^{-0.4}$

$y^{-\frac{2}{5}} = \frac{1}{y^{2/5}} = \boxed{\frac{1}{\sqrt[5]{y^2}}}$

3. Write the radical expression in exponential form.

a) $\sqrt[4]{c^3} = \boxed{c^{\frac{3}{4}}}$

b) $\left(\sqrt[3]{b}\right)^5 = \boxed{b^{\frac{5}{3}}}$

4. Simplify.

a) $25^{-2.5}$

$25^{-\frac{5}{2}} = \frac{1}{25^{5/2}}$

$\frac{1}{(\sqrt{25})^5} = \frac{1}{5^5} = \boxed{\frac{1}{3125}}$

b) $\left(243a^{-10}\right)^{\frac{2}{5}}$

$243^{2/5} \quad a^{-10 \cdot \frac{2}{5}}$

$(\sqrt[5]{243})^2 \cdot a^{-4}$

$(3)^2 a^{-4} = \boxed{\frac{9}{a^4}}$

Assignment:

Day 1: pg 388 (1-26 all, 30-37)