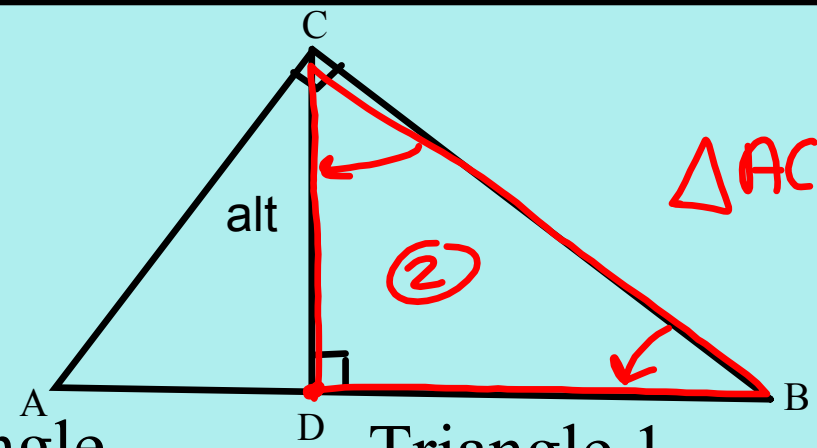
Geometry

Ch. 7 Handout 7.4

Similarity in Right Triangles

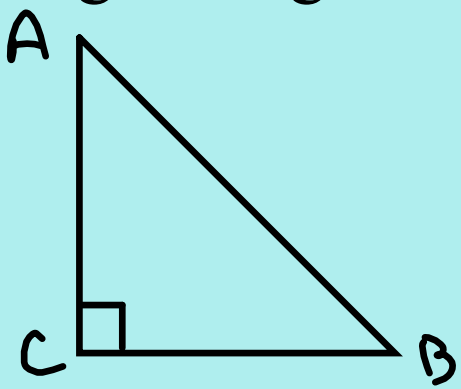
Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

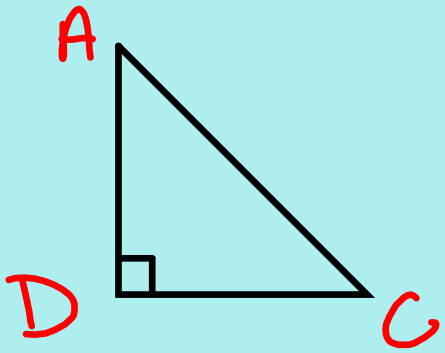


$\triangle ACB \sim \triangle ADC \sim \triangle CDB$

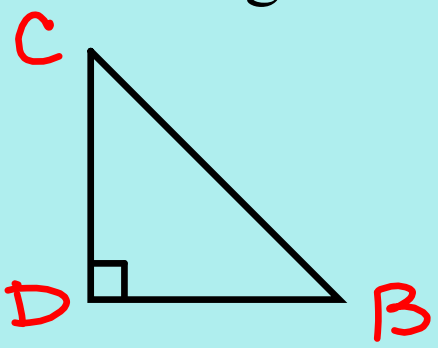
Big Triangle



Triangle 1



Triangle 2



Geometric Mean

Pull

is the positive number x such that

$$\frac{a}{x} = \frac{x}{b}$$

means-same

extremes

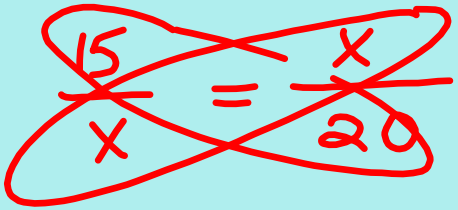
Ex. 1 Find the geometric mean of 3 and 12.

$$\frac{3}{x} = \frac{x}{12}$$

$$\sqrt{36} = \sqrt{x^2}$$

$$x = 6$$

Ex. 2 Find the geometric mean of 15 and 20.



$\sqrt{300} = \sqrt{x^2}$

$x = \sqrt{300}$

$= \sqrt{100 \cdot 3}$

$\boxed{x = 10\sqrt{3}}$

8 is the geometric mean between a number and 2.

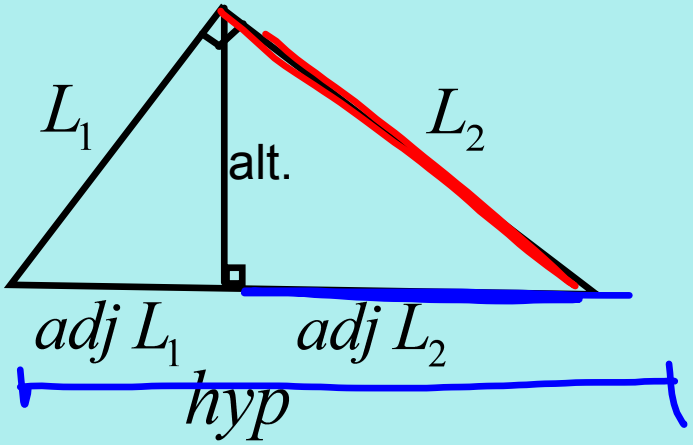
$$\frac{x}{8} = \frac{8}{2}$$

$2x = 64$

$x = 32$

Corollary 1 to Theorem 7-3
The lengths of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

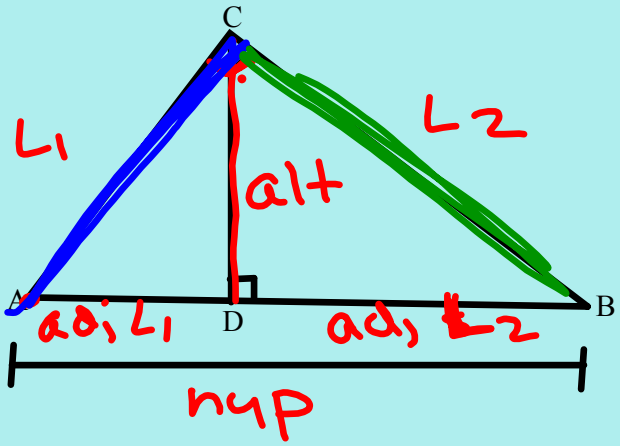
Corollary 2 to Theorem 7.3
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.



$$\frac{\text{adj } L_1}{\text{alt}} = \frac{\text{alt}}{\text{adj } L_2}$$

$$\frac{\text{adj } L_1}{L_1} = \frac{L_1}{\text{hyp}}$$

$$\frac{\text{adj } L_2}{L_2} = \frac{L_2}{\text{hyp}}$$



alt leg 1

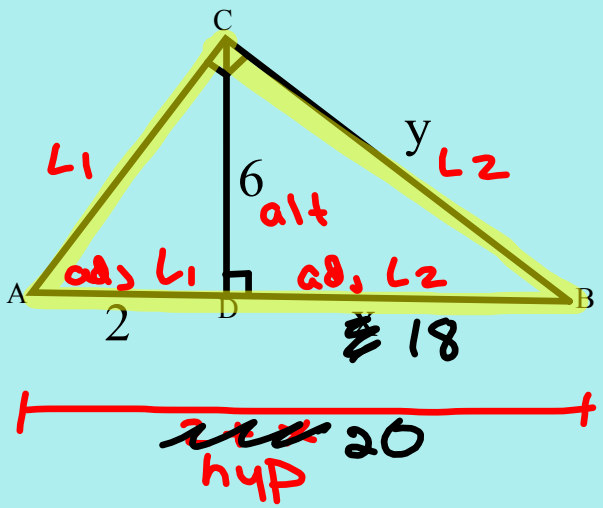
hyp leg 2

adj. seg leg 1

adj. seg. leg 2

$$\frac{\text{adj. seg leg 1}}{\text{alt}} = \frac{\text{alt}}{\text{adj. seg. leg 2}} \qquad \frac{\text{adj. seg leg 1}}{\text{leg 1}} = \frac{\text{leg 1}}{\text{hyp}} \qquad \frac{\text{adj. seg. leg 2}}{\text{leg 2}} = \frac{\text{leg 2}}{\text{hyp}}$$

Ex. 3 Solve for x and y.



$$\frac{\text{adj } L_1}{\text{alt}} = \frac{\text{alt}}{\text{adj } L_2}$$

$$\frac{2}{6} = \frac{6}{x}$$

$$2x = 36$$

$$\boxed{x = 18}$$

$$\frac{\text{adj } L_2}{L_2} = \frac{L_2}{\text{hyp}}$$

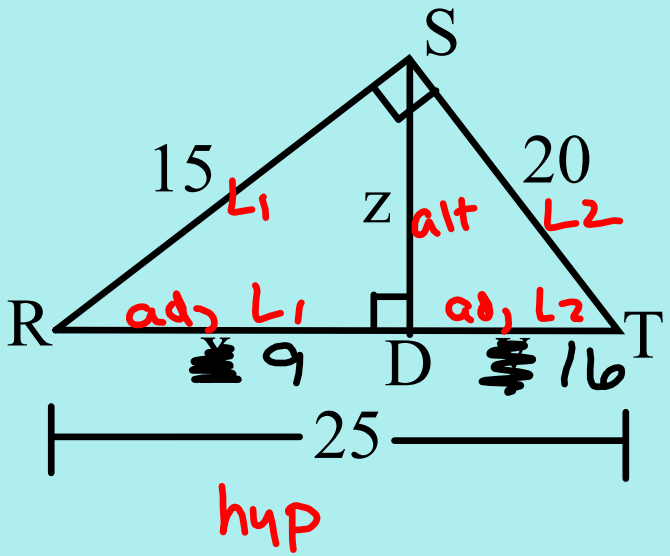
$$\frac{18}{y} = \frac{y}{20}$$

$$\sqrt{y^2} = \sqrt{360}$$

$$y = \sqrt{36 \cdot 10}$$

$$\boxed{y = 6\sqrt{10}}$$

Ex. 4. Solve for x, y, and z.



$$\frac{adj\ L_1}{L_1} = \frac{L_1}{hyp}$$

$$\frac{x}{15} = \frac{15}{25}$$

$$25x = 225$$

$$x = 9$$

$$y = 16$$

$$\frac{adj\ L_1}{alt} = \frac{alt}{adj\ L_2}$$

$$\frac{9}{z} = \frac{z}{16}$$

$$\sqrt{z^2} = \sqrt{144}$$

$$z = 12$$

$$z = 12$$

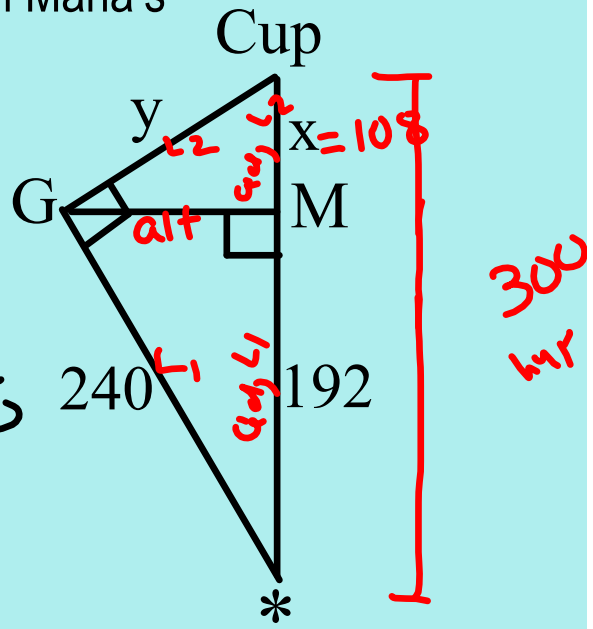
Ex. 6: At a gold course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, their remaining distances from the cup. Find the distance between Maria's ball and Gabriels's ball.

$\frac{\text{adj } L_1}{L_1} = \frac{L_1}{\text{hyp}}$ $\quad \frac{\text{adj } L_2}{L_2} = \frac{L_2}{\text{hyp}}$

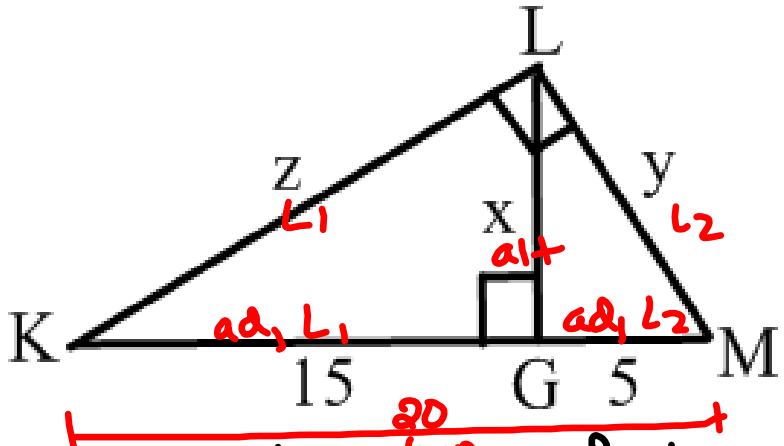
$\frac{192}{240} = \frac{240}{z}$ $\quad \frac{108}{y} = \frac{y}{300}$

$192z = 57{,}600$ $\quad \sqrt{y^2} = \sqrt{32{,}400}$

$z = 300$ $\quad y = 180$



Ex. 7 Solve for x, y, w, and z.



$y = 10$

$z = 10\sqrt{3}$

$x = 5\sqrt{3}$

$\frac{adj\ L_1}{alt} = \frac{alt}{adj\ L_2}$ $\quad \frac{15}{x} = \frac{x}{5}$ $\quad x^2 = 75$ $\quad x = \sqrt{25 \cdot 3}$ $\quad x = 5\sqrt{3}$

$\frac{adj\ L_1}{L_1} = \frac{L_1}{hyp}$ $\quad \frac{15}{z} = \frac{z}{20}$ $\quad z^2 = 300$ $\quad z = \sqrt{100 \cdot 3}$ $\quad z = 10\sqrt{3}$

$\frac{adj\ L_2}{L_2} = \frac{L_2}{hyp}$ $\quad \frac{5}{y} = \frac{y}{20}$ $\quad y^2 = 100$ $\quad y = 10$