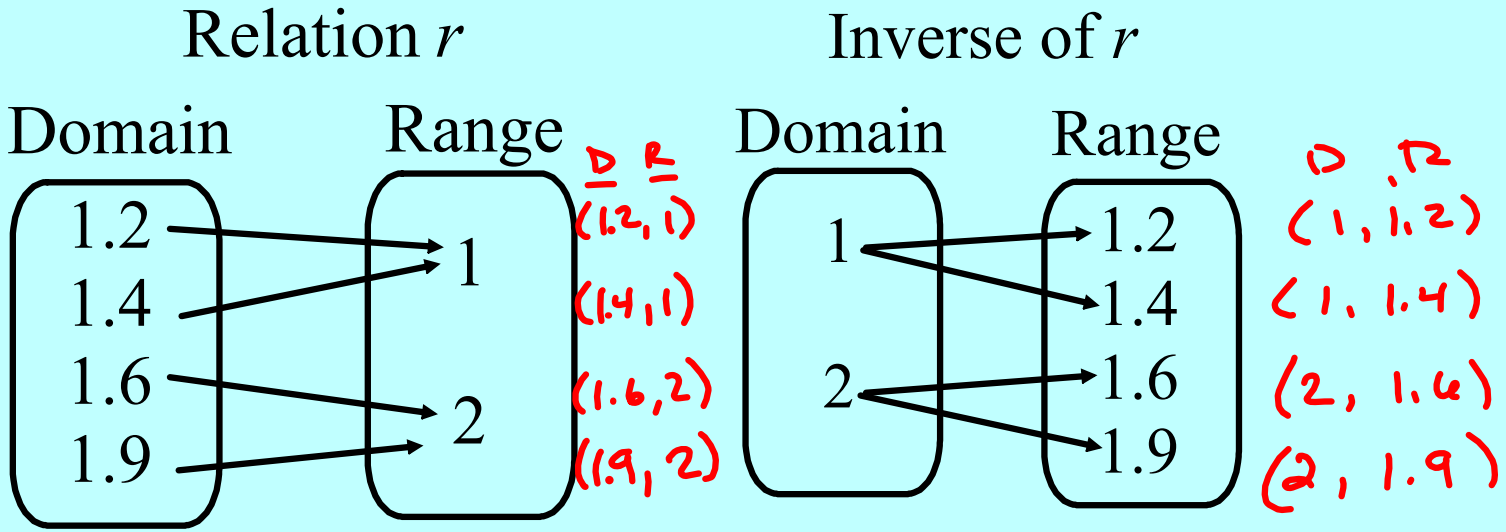
Algebra 2

Ch. 7 Handout 7.7

Inverse Relations and Functions

If a relation maps element *a* of its domain to element of *b* of its range, the inverse relation "undoes" the relation and maps *b* back to *a*. So, if (a, b) is an ordered pair of a relation then (b, a) is an ordered pair of its inverse.



The inverse of function *f* is denoted by $f^{-1}(x)$. If a function *f* pairs *a* with *b*, then $f^{-1}(x)$ must pairs *b* with *a*.

The range of the relation is the domain of the inverse, and the domain of the relation is the range of the inverse.

Function → Inverse Function

(a , b) → (b , a)

Example 1: Finding the Inverse of a Relation --

a) Find the inverse of the relation.

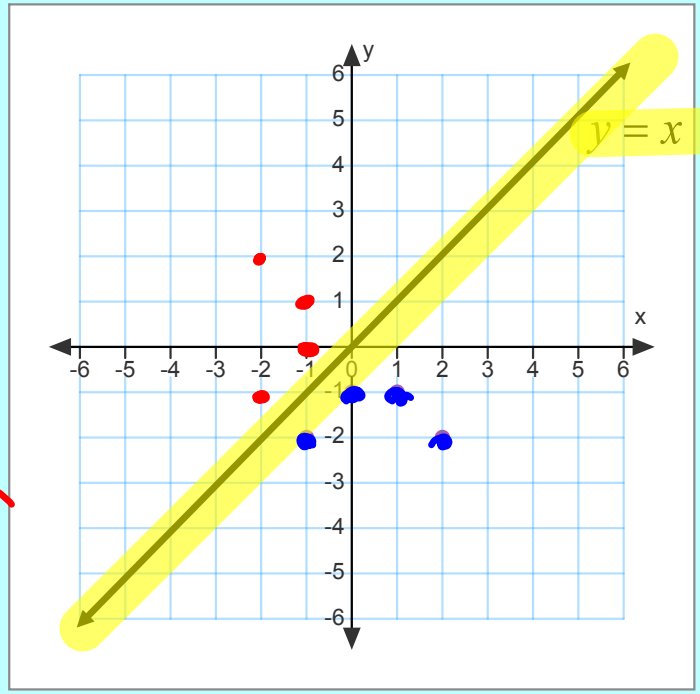
Relation m function

d	x	-1	0	1	2
r	y	-2	-1	-1	-2

Inverse of Relation m

d	x	-2	-1	-1	-2
r	y	-1	0	1	2

Not a function



Finding Domain: Two Rules: a) den $\neq$ zero
b) $\sqrt{\ } \geq 0$

→ value for x

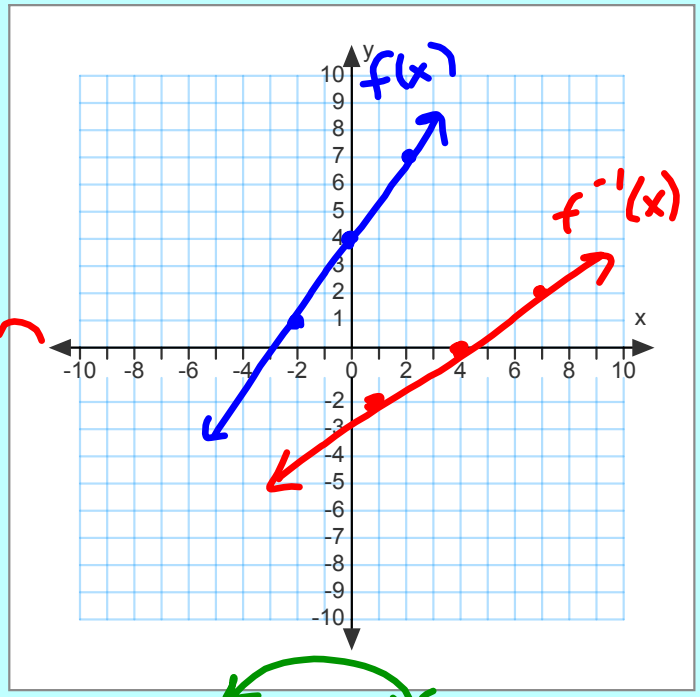
To find the inverse relation.
Interchange x and y.
Solve for y.

Does $y = x^2 - 2$ define a function?

Is its inverse a function?

Ex 3: Graph each relation and its inverse. Find the domain and range of the relation and its inverse. Is the inverse a function?

$$y = \frac{3}{2}x + 4$$



Function

$y = \frac{3}{2}x + 4$

x	y
2	7
0	4
-2	1

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Function

Inverse Relation

$y = \frac{2}{3}x - \frac{8}{3}$

x	y
7	2
4	0
1	-2

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Function

$y = \frac{3}{2}x + 4$

$x = \frac{3}{2}y + 4$

$-4 \quad -4$

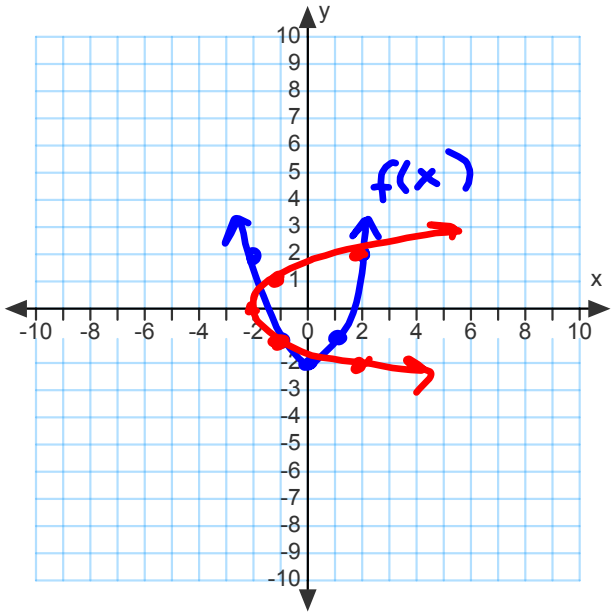
$(2)\ x - 4 = \frac{3}{2}y\ (2)$

$2x - 8 = 3y$

$\frac{2}{3}x - \frac{8}{3} = y$

Ex 4: Graph each relation and its inverse. Find the domain and range of the relation and its inverse. Is the inverse a function?

$$y = x^2 - 2$$



Relation

$y = x^2 - 2$

x	y
-2	2
-1	-1
0	-2
1	-1
2	2

D: $(-\infty, \infty)$

R: $[-2, \infty)$

Function

Inverse Relation

$y = \pm\sqrt{x+2}$

x	y
2	-2
-1	-1
-2	0
-1	1
2	2

D: $[-2, \infty)$

R: $(-\infty, \infty)$

Not a function

$y = x^2 - 2$

$x = y^2 - 2$

$\pm\sqrt{x+2} = \sqrt{y^2}$

$y = \pm\sqrt{x+2}$

Ex. 5: For each function f, and f^{-1}, the domain and range of f and f^{-1}, and determine whether f^{-1} is a function.

$f(x) = \sqrt{2x+2}$

function

$y = \sqrt{2x+2}$

Find Domain 1st:

$2x+2 \geq 0$

$2x \geq -2$

$x \geq -1$

x	y
-1	0
0	$\sqrt{2} \approx 1.4$
1	2
2	$\sqrt{6} \approx 2.4$
4	$\sqrt{10} \approx 3.2$

D: $x \geq -1$ or $[-1, \infty)$

R: $y \geq 0$ or $[0, \infty)$

function

Inverse

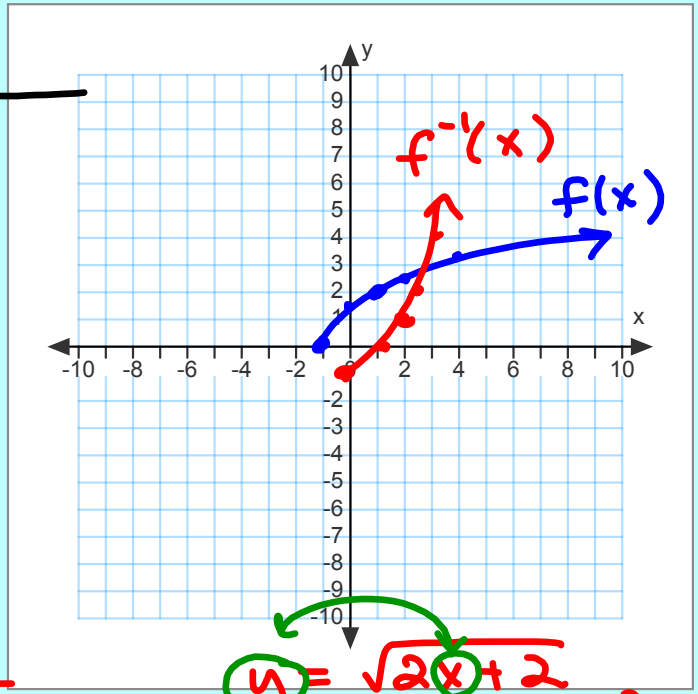
$y = \frac{1}{2}x^2 - 1$

x	y
0	-1
1.4	0
2	1
2.4	2
3.2	4

D: $[0, \infty)$

R: $[-1, \infty)$

function



$y = \sqrt{2x+2}$

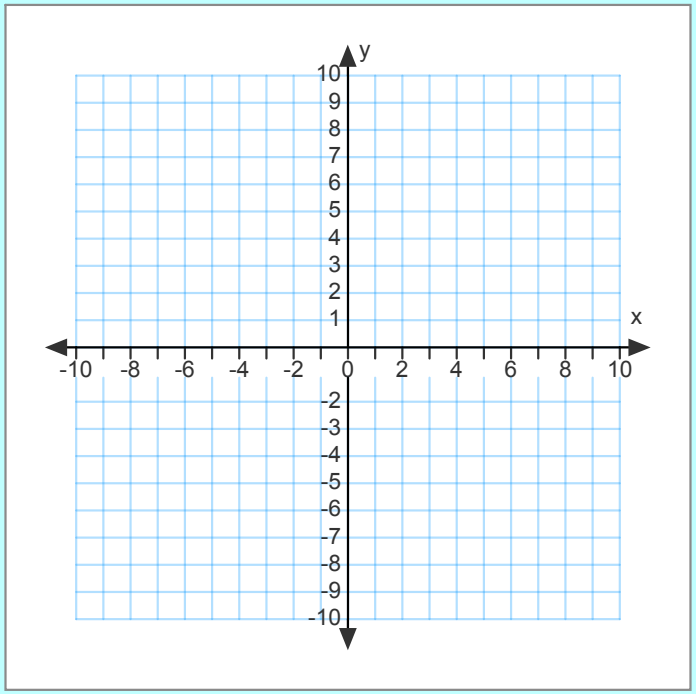
$(x)^2 = (\sqrt{2y+2})^2$

$x^2 = 2y + 2$

$\frac{x^2}{2} - \frac{2}{2} = \frac{2y}{2}$

Ex. 6: For each function f, and f^{-1}, the domain and range of f and f^{-1}, and determine whether f^{-1} is a function.

$$f(x) = 2x^2 - 1$$

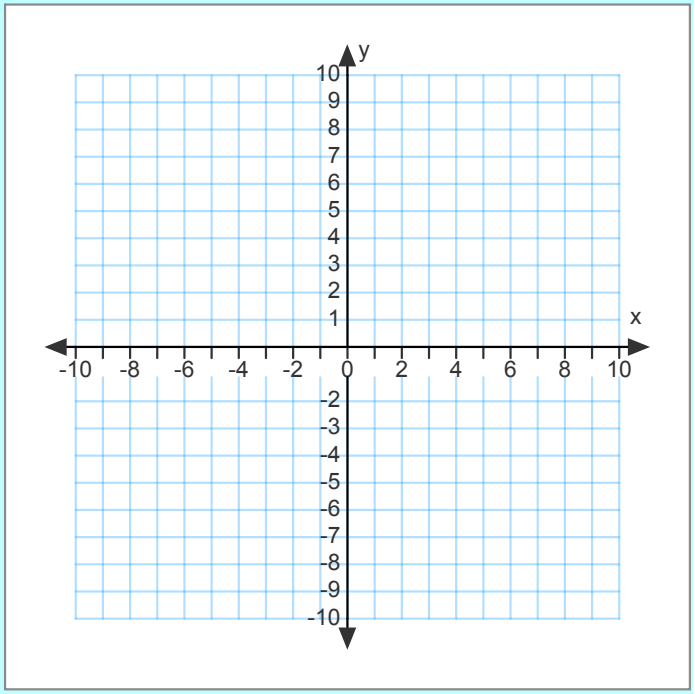


Find the inverse of each function.
Is the inverse a function?

(11) $y = (x+1)^2$

Ex. 7: For each function f, and f^{-1}, the domain and range of f and f^{-1}, and determine whether f^{-1} is a function.

$$f(x) = \sqrt{x+9}$$



Functions that model real-life situations are frequently expressed as formulas with letters that remind you of the variables they represent. When finding the inverse of a formula, it would be very confusing to interchange the letters. Keep the letters the same and just solve the formula for the other variable.

Example 8: The function $d = 16t^2$ models the distance d in feet that an object falls in t seconds. Find the inverse of the function. Use the inverse to estimate the time it takes an object to fall 50 ft.

Assignments:

Day 1: pg 410 (1-21 odds -- #'s 15-21 find domain and range of relation and inverse)